DIDACTIC GAMES IN MATHEMATICS

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PREFACE

Children's games set up their
whole future lives; their personality,
dispositions and innermost
tendencies are expressed in and
developed through games.

F. W. A. Fröbel, (1826)

A game is an activity which is attractive for most pupils and also for the majority of the adult population. Currently **gamification** is becoming increasingly popular. Gamification is the use of game approaches and mechanisms in order to increase the involvement and to support the desired actions of people in various non-game contexts. Teachers have long tried to use the attractiveness of games and their motivational potential in teaching in the form of **didactic games**. The aim of this publication is to present didactic games and their use in mathematics teaching at both elementary and secondary schools. It contains a **theoretical part** related to didactic games, the **methodology** of their use during lessons, and a **collection of didactic games**.

We believe that the publication will meet its purpose, and it will be both educational and engaging.

Peter Vankúš
Bratislava, 2013

INTRODUCTION

The subject of mathematics is so
serious that we should not ignore
any opportunity to make it
interesting.
B. Pascal, (1914)

This publication comprises 6 chapters dealing with the issue of didactic games in mathematics from different perspectives.

In chapter I you will find a description of the notion of 'game' and a definition of a didactic game.

Chapter II contains a historic review of opinions about the aim of games in education. It includes the opinions of many historic characters that have supported the role of games in education.

Chapter III covers the research of using didactic games in mathematics teaching. The outcomes of such research confirm the rightfulness of integrating didactic games into mathematics education.

Chapter IV describes the selection of suitable didactic games, and the methodology of using didactic games in the teaching process.

Chapter V is a collection of selected didactic games suitable for meeting educational targets in mathematics teaching, determined for classes 5–9 of elementary and secondary school.

Chapter VI is a table where didactic games are classified depending on the mathematical areas that the games are primarily useful for.

The first three chapters are important for obtaining a theoretical overview of the issue of didactic games. Chapters IV, V and VI are important for a practical use of didactic games, while the first three chapters can also be studied subsequently.

1 DIDACTIC GAMES

Playing and games are some of
the most important elements of
children's development, as they are
a natural expression of children's
inner needs.

F. W. A. Fröbel, (1826)

The key term in this work is **didactic game**. We will try to describe this term in more detail.

In any language, the word 'game' has unusually numerous meanings. It is used to describe fun and entertainment, but also a theatre or music piece, or a careless activity, e.g. 'play a hero'.

We can also see that different nations have a different understanding of the word 'game'. In ancient Greece the word 'game' meant activities specific for children, mainly what we denote today as frolicking; while the Jewish understood the word 'game' as bantering and laughing. The Roman 'ludo' meant a game, joy and cheerfulness; in Sanskrit 'kliada' meant a game, pleasure. The current European languages under 'game' understand a wide area of human activities which are not related to hard work on one hand, and provide joy and satisfaction to people on the other. And so many notions have been included under this expression, starting from children playing soldiers up to representing heroes on

theatre stages, from a contemplative chess game, up to a violinist's master art. (**Eľkonin, 1983**)

Looking in a pedagogic dictionary (**Průcha, Walterová** and **Mareš, 1998, p 82**) we will find the following under 'game':

Game: A form of activity, different from work or study. People have been dealing with 'games' all their lives, but games have a specific role at their pre-school age – it is the main type of activity. A game has numerous aspects: cognitive, practicing, emotional, motional, motivational, creative, fantasy, social, recreational, diagnostic and therapeutic. It involves the activities of an individual, a pair, a small or a big group. There are games which require special aids (toys, game instruments, sport equipment, tools, devices). Most games have the form of social interaction with explicitly formulated rules (given by the agreement of the persons involved or by conventions). Much attention is paid to the course of a game (games with a focus on cooperation, competition). It is possible to formalize a starting situation, the course, and results of some games, and to study the decision-making of players in detail. A special discipline of mathematics deals with these issues – the game theory.

The mentioned definition contains the differentiation of a game from work or study. It mainly lies in the fact that participation in a game is not obligatory, and contrary to work, material remuneration is not an impulse to play. Participation in a game is mainly motivated by positive

feelings and the joy of its course. Importantly, certain aspects that are also necessary for work or study can be developed through games. The following definition of a 'game' (**Böhm, 1988, pp. 556–557**), which is identical in the meaning of this word with the previous understanding, contains brief opinions on the function of a game:

Game: A spontaneous activity the purpose and target of which is the game itself, and it does not consciously pursue other targets (contrary to work). Each human activity may acquire the nature of a game when performed just for fun, and for the pleasure of performing it. So there are: empirical, emotional, imaginative, imitating, language, social, love, mental and other games. There are always attempts to divide the manifoldness of games and to clarify them according to their functions. The following approaches to games are the most common:

- *a game as an expression of excess energy (Spencer);*
- *understanding a game as a return to previous stages of development (Stanley, Hall);*
- *relaxation (Carr, Freud, Adler);*
- *unconscious preparation for the future (Groos, Claparede);*
- *form of cognition (Volpicelli, Fink).*

In addition to didactic functionality and the related engagement of senses (e.g. educational and didactic games), a game has deep pedagogic-anthropological reasoning according to Fröbel and Schiller. Fröbel thinks that a game as an "activity oriented from the inside of an individual outwards,

while at the same time the individual is receiving external stimuli inwards" exceeds the one-sided focus of study and work, and leads to the symbolic cognition and representation of the world. Schiller sees a form of human life in a game. He thinks that through a game people not only fulfil their instincts and desires, but also deprive the surrounding conditions and binding obligations and rules, and have a nicer life.

As it is clear from the definition that functions directly connected to education are attributed to games: preparation for the future, a form of getting to know the world. **J. Piaget** and **B. Inhelder**, who considered the function of games in the development of children's psychology in detail, attributed a biological function of active repetition and experimentation to a game through which new situations and experiences are mentally processed and prepared (**Piaget** and **Inhelder, 1997**). Considering its functions, a game seems to be a natural way of obtaining knowledge and acquiring new mental processes. It is thus understandable that it has also become part of school teaching. In his *Laws*, **Plato** recommends giving apples to children to play with when teaching them arithmetic; he recommends suitable building toys to educate future builders. The appeal of **J. A. Comenius** 'school through play' is well-known. Certain educational theories consider a game as one of the main teaching methods (e.g. Progressivism, Waldorf education, French Group of New Education GFEN). For more information about

opinions regarding the educational role of games, see the chapter *Games in the History of Education*.

In order to specify the characteristics of a game as a didactic method, we commonly use the term a *didactic game.* Under 'didactic game' we understand a game with rules, fulfilling a certain didactic target (**Kárová, 1996; Foltinová** and **Novotná, 1997**). Its main differences from a spontaneous children's game are:

- compulsory participation of pupils;
- its use to achieve certain educational targets;
- external game management (often through rules).

The pedagogic dictionary (**Průcha, Walterová** and **Mareš, 1998, p. 48**) describes and characterizes a didactic game:

Didactic game: Analogy to a spontaneous children's activity pursuing didactic targets (not always in an apparent way for pupils). It may take place in a classroom, gym, playground or outdoors. It has its own rules, and requires constant supervision and final assessment. It is meant for both individuals and groups of pupils, while the role of a pedagogic leader has a wide scope: from a main organizer up to an observer. Its priority is its stimulating nature, as it stirs interest, increases the engagement of pupils in the performed activities, stimulates their creativity, spontaneity, cooperation and competitiveness, makes them use different knowledge and abilities, and engages their life experience. Some didactic games simulate real-life situations.

Let's look closer at this definition. A didactic game is an activity for both pupils and a teacher focused on achieving certain didactic targets. Let's compare this statement with the definition of a teaching method understood as 'arrangement of learning stuff and the activities of the teacher and the pupils, in order to achieve determined didactic targets,' (**Stračár, 1979; Pavlík et al., 1984; Zelina** and **Nelešovská, 1983**). We can see that a didactic game can be considered as a teaching method. **L. Mojžíšek** mentioned a game separately as a teaching method in his overview of teaching methods in1975.

The above-mentioned definition also delimits a certain standard structure of a didactic game. The most important parts of a didactic game are:

- game environment;
- game targets;
- the game procedure itself, determined by rules;
- final game assessment.

The game environment is to be understood as the material environment: necessary aids and equipment. Another component of this environment is the game itself, its rules, task assignments, the procedure, and the form of activities of both pupils and the teacher. Naturally, the most important part of the game environment is the pupils and teachers involved. Pupils bring their expectations, experience, and attitudes to mathematics, as well as their knowledge and skills to each game. The teacher usually has a controlling and

organizational function. His/her task is to ensure the smooth and successful course of a game. The arrangement of the environment should motivate the pupils, and encourage them to actively participate in the game and try to achieve the game targets.

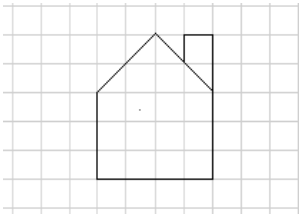
Didactic game targets are determined by an educational target which we want to achieve through the game. On the basis of the given target, we choose a suitable type and form of didactic game. The use of a game as a teaching method is only meaningful when it enables the achievement of the determined educational targets.

The game procedure itself is a performance of a didactic game through the activities of pupils and the teacher. It is necessary that this activity is interesting for pupils and motivates them to be active. It must be appropriate for their age and skills, and respect their needs. At the same time, it must be focused on achieving the educational target. Game rules ensure that the work leads to the achievement of the game purpose. **Game rules** determine the character and form of pupils' activities, they organize their activity. Game elements such as competitiveness or an effort to achieve better results are usually hidden in the rules.

Final game assessment verifies the achievement of the educational target, its task is to reward pupils and motivate them for other activities.

To make it clearer, we will present a particular example of a didactic game:

The curriculum of the 5th class of elementary schools – *Area of geometric shapes (rectangle, square),* also contains the following subject matter: *Area of geometric shapes in a square grid*. In exercises related to this subject matter, pupils need to determine areas of simple shapes in a square grid (*Picture 1.1*). We will decide to use a didactic game for the pupils to help them automatize the given subject matter. The aim of this game is to practice the given subject matter by solving tasks and feedback about the quality of knowledge both for the pupils and the teacher. After determining the given target, the level of pupils' knowledge and the character of the subject matter, we will choose a suitable didactic game.



Pic. 1.1 The task is to determine the area of geometric shapes (one square in the grid has an area of 1 cm^2)

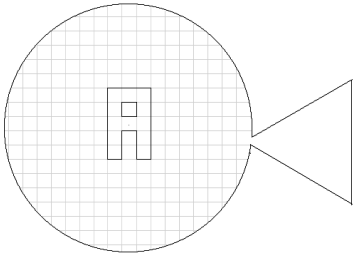
Game name: Mathematical fishing (**Foltinová** and **Novotná, 1997**)

Game environment:

Pupils and the teacher: The class is divided into teams of 5 pupils. The teacher has an organizational and controlling role.

Material environment: Task cards with corresponding point scores (10 points, 5 points, they may have the shape of fish – see *Picture 1.2*) for each team.

Game duration: 10–15 min



Pic. 1.2 Game card for the *Mathematical fishing* game

Game procedure:

The teacher distributes the cards into two groups, depending on difficulty (cards of different difficulty have different colours). The pupils will determine the area of the shapes in the cards. The pupils select the difficulty themselves by choosing which group they will choose a task from. If they succeed in solving the task, their team wins the corresponding number of points. The teacher assesses the correctness of the task solutions, and records the scores of single teams. Tasks are numbered to enable faster control, while the teacher has the matching results to each number. Pupils put down the solutions on a special piece of paper, after a certain time (10–15 min) the results are checked. The task of the teams is to obtain the highest number of points.

Final assessment:

After the end of the game, the scores of single teams are counted, and a ranking of teams is made on the basis of the obtained numbers of points. Pupils are rewarded with points for activity – pupils of the winning team obtain the highest number of points, pupils of the team ranked last obtain the lowest, but not zero, number of points.

Advantages of the game:

The differentiated difficulty of tasks; inner motivation of pupils through competitiveness among groups; weaker pupils may also contribute to the team's success; active work of the class; pupils work in a context attractive for them; feedback about the level of pupils' knowledge in single groups; pupils develop their social skills through their interaction in teams.

On the basis of the previously mentioned understandings of a 'didactic game', we have formulated our working definition of this term:

A didactic game is understood as an activity of the pupils and teacher which pursues certain didactic targets. Pupils usually do not realize these targets. The motivation of their activity is the joy of performing it, competitiveness, the opportunity to work in favour of a team, self-fulfilment... A didactic game has rules that organize the pupils' activity. This activity, its contents, and didactic game rules help to achieve the educational targets of the game. A didactic game is characterized by the high involvement and motivation of pupils, and the pleasure of performing a playful activity.

2 GAMES IN THE HISTORY OF EDUCATION

History is the witness of the times, the torch of truth, the life of memory, the teacher of life, the messenger of antiquity.
M. T. Cicero, (2001)

The history of games and their use in education and upbringing started long ago. In this chapter we will describe the opinions of important characters regarding the role of games in education. The chapter is elaborated according to the article *The history and present of didactical games as a method of teaching mathematics* (**Vankúš, 2005**).

Even the ancient Greeks recommended games as a means of education. **Plato** (427 BC – 347 BC) in his works *Laws* and *Constitution* justified the use of games in education. According to Plato, the main educational method for children aged 3–7 is a game. Educational games were meant to prepare children for their future work activities. Plato thinks, for example, that manipulation with jigsaws is a suitable part

of the education of future builders. Natural children's talents will be manifested in such playful activities (**Plato, 1980, p. 310**):

...teach boys about these disciplines not by violence, but in a playful way, and at the same time, you can also find out what each of them has a natural inclination for.

Plato's scholar **Aristotle** (384 BC – 322 BC) also realized the importance of games for education. In his works *Politics* and *Nicomachean Ethics*, he defended the need for games at children's age. He thinks that a suitable game is the most appropriate activity for children.

Schooling in ancient Rome continued in the Greek educational tradition. The first schools appeared here at the start of the republican establishment, and they were called 'ludi', i.e. games. Most of the games used in these schools were related to the physical development of pupils.

After the fall of the Western Roman Empire, a feudal social establishment was formed in European countries in the 5th–10th centuries. It flourished during the 11th–15th centuries, while medieval Christian religion and the church had great influence on education during this period. Education was aimed at developing humility and the obedience of pupils. Hard discipline was therefore applied at schools, bodily punishments were used. The main teaching method was drilling. There was little scope for using didactic games in this form of teaching.

This does not mean, however, that games did not have an important place in the lives of children in the Middle Ages. In archaeological collections dating from the 14th to 15th centuries, we can find dolls, figurines of dogs and knights on horses, various ceramic toys... According to the findings, the most popular toy on the territory of Slovakia until the 16th century was a rocking horse. So also at that time, a game was a preparation for future life activities: the rocking horse was a preparation for a "knight's life", dolls were a preparation for girls as future mothers, various wooden and clay tools helped to form work skills and habits. However, the systematic use of games in school education was tabooed in the period of feudalism.

It was the Renaissance which brought about better times for the use of games in education. Instead of humility and obedience, the education of a physically developed and intellectually educated human became the centre of interest for education again. Due to its orientation on a human, this movement is called Humanism. Humanists criticised medieval schooling very sharply. They refused the upsetting verbalism and formalism in teaching, and rejected the severe discipline of medieval schools. They required the ancient Greek and Roman authors to be studied at schools. They wanted the following sciences to be taught: natural sciences, geography, physics, but also history. These humanistic requirements and attitudes resulted in the need to change educational methods. Teaching should be demonstrative, to

stir pupils' activity. The life needs and interests of pupils were also to be considered. Games were also used as a way to fulfil these criteria.

Some teachers in the 15th to 18th centuries shared the positive ancient Greek and Roman attitude towards the games. The interest in using games as a significant educational method was also supported by the extensive production of toys in Europe, mainly concentrated in Germany. This is exactly where **J. A. Comenius** (1592–1670), the great European educator, studied. According to his opinions, a game is a very important element of upbringing, as education itself should be playful and joyful. It is therefore, necessary to constantly arouse the interest of pupils in teaching and to explain subject matter so that it has the nature of a game.

Comenius laid the foundations for the 'school through play' in his works *Informatórium školy materskej (Book of Nursery-school Teachers)*, *Škola hrou aneb živá encyklopedie (School by play or a life encyclopaedia)* and *Vševýchova (All-education)* (**Comenius, 1957, 1959**). He understood games, as a natural way to foster a child's will and character features, as a spontaneous expression of a child's activity bringing pleasure. In the education of the youngest children, he emphasizes (**Comenius, 1957, pp. 137–138**):

Parents should try to ensure that their children have joy in their lives. To summarize, when you see whatever the child likes and welcomes, it should not be denied to him/her; on the

contrary, if you see that a toy is nice to his/her eyes, ears and other senses, it will be refreshing for the child's body and mind.

Comenius also stressed that children need to play group games to develop their social skills. In his opinion, games have a beneficial effect on the health of children and they develop their senses, memory, judgement, work fervour and eloquence. Comenius emphasizes the use of bodily games for the physical development of pupils. He also acknowledged the function of games as a preparation for future work. He stressed the need to lead children from a spontaneous playful activity to intentional work – in this sense he pointed out the guiding of children during games and the use of games with rules.

Humanistic opinions about education appear in the pedagogic work of the English philosopher, scholar and pedagogue **J. Locke** (1632–1704). Locke dealt with the issues of upbringing and education from both the theoretical and practical points of view. He criticises medieval schools and their methods. He thinks that teaching should be performed naturally and not by force. When teaching reading and writing, Locke recommends using games with letters and picture books. (**Locke, 1959**)

The work of the French enlightener **J. J. Rousseau** (1712–1778) *Emile or On Education* also comes from the Renaissance. In it the author defends the humanistic opinions of childhood. Until the Renaissance, the age of 6–7

was considered to be the age limit of childhood. After reaching this age, children were fully integrated into the work process. The right of children to a longer 'playful' childhood started to be more accepted during the Renaissance.

In his work, Rousseau demands that education is performed in natural ways with regard to age particularities; he refused the suppression of pupils' personalities, dull cramming of subject matter, or excessive discipline. According to Rousseau, the basis of education and upbringing should be a child's own observations, contemplation and personal experience. Naturally the use of games was appropriate here, as an adequate and free activity for children. Rousseau himself says (**Rousseau, 1910, p. 81**):

Love children, let them have their games, fun, and their lovely instinct.

Particularly, Rousseau paid attention to motional games, games for the development of writing, counting and musical talent. He formulated rules which need to be followed during games: appropriateness of the game for children, its joyful and non-enforced behaviour. According to Rousseau, a game is a child's natural activity which best satisfies their need for activity.

The German pedagogue **J. H. Pestalozzi** (1746–1827) further developed Rousseau's ideas. He placed importance on learning through activity. Through this activity, he is trying to encourage the pupil to display their interests. One such activity was a game, in his opinion. He pointed out the need

for the systematic use of games to achieve educational goals. (**Biber, 1831**)

The first research in the area of games and their educational function date back to the 18th century. A pedagogic system of games prepared by the German priest and pedagogue **F. W. A. Fröbel** (1782–1852) is well-known. Fröbel was Pestalozzi's scholar and follower. He also believed in the great importance of games in education and upbringing, and recommended their use in children's education. He thinks that games are a means of a healthy evolution of young people. His following statement confirms this (according to **Lange, 1863, p. 33**):

Playing and games are the highest level of children's development, the development of a person of this age, as they are a natural expression of children's interior, an expression which originates from the internal need of the child itself.

Fröbel divides children's evolution into 3 periods: a suckling, a child, and an adolescent. In the 'suckling period' children's senses develop, 'childhood period' is the time of games and speech development, while the 'adolescent period' is the period of schooling when educational goals can be consciously achieved. We can see the importance which Fröbel placed on games in pre-school education in the following quotation (according to **Lange, 1862, p. 469**):

Games should serve children as an all-round and joyful practice, all their powers and abilities should be educated in

clear joy; and children should live in harmony and holy childishness and prepare for school and their future life.

Fröbel revised in detail and tried the methodology of education in pre-school establishments ("Kindergartens") as well as a set of toys suitable for the education of children of this age. This set of children's toys is known as "Fröbel's gifts".

It is a collection of 6 sets of simple toys. The first gift is a set of 7 cotton balls of different colours, fixed to a thread. They should help children to recognize colours and master spatial imagination (e.g. the terms "forward" and "backward" related to the movement of balls). The second gift is a wooden sphere, cube and cylinder. The purpose of this gift is to make children familiar with basic geometric shapes. The third gift is a cube divided into 8 cubes; the fourth gift is a cube divided into 8 boards. The fifth and sixth gifts are cubes divided into 27 small cubes, some of which are further sub-divided. All these divided cubes should teach children to build shapes and thus develop their imagination and combination skills. Fröbel elaborated an exact methodology for the use of his gifts, he established a set of accompanying words, songs and movements. His gifts became widespread (**Váňa et al., 1958, p. 112**).

Fröbel's view of the educational use of games can be expressed in the following quotation **(Fröbel, 1826**):

Children's games set up their whole future lives; their personality, dispositions and innermost tendencies are

expressed in and develop through games. The whole future life of a man has its source in his childhood.

Renowned psychologists, philosophers and pedagogues made detailed analyses of games and their importance in the lives of people in the 19th century. Let's mention J. W. Goethe, F. Schiller, H. Spencer and K. Groos from among many others.

J. W. Goethe (1749–1832), the important German playwright, pointed out the importance of games in education. He was particularly interested in the imaginative and dramatic elements of games. He thought that games developed imagination, memory and emotivity. Goethe understood a game as a means of general personal development (**Cheyne, 1989, p. 25**).

F. Schiller (1759–1805), the German poet and philosopher, reflected his ideals of freedom and sense in games. He considers them to be a form of activity, enabling individuals to express themselves freely and thus to achieve a more beautiful life. Schiller contemplated the reasons for playful behaviour. According to him, the play of animals and people is an expression of excessive life energy which nature has endowed them with generously and which has not been used in the fight to survive. (**Schiller, 2004**)

H. Spencer (1820–1903), the English philosopher, sociologist and pedagogue, advocated such education and upbringing which are aimed at preparation for life. In his work (**Spencer, 1911**) he determined didactic principles

which have become characteristic of Anglo-Saxon pedagogic thinking. These principles contain the requirement of active and joyful learning based on pupils' experience. Spencer emphasized the need for sufficient games for pupils.

He also contemplated about why animals and people play. In his opinion, games are an expression of the need to relieve excessive energy (**Spencer, 1911**):

A game is a substitute application of energy which has not been used where it was naturally supposed to be used, and thus it has accumulated to such an extent that it needs to be relieved and expressed with a playful activity.

K. Groos (1861–1946), the German psychologist and pedagogue, elaborated the first comprehensive concept of the reasons and meanings of playful behaviour. In his works, (**Groos, 1896, 1899**) he emphasizes the function of games as a preparation for adult tasks and behaviour. In the first of the mentioned works, Groos wrote:

We cannot think that animal play is a luxury of youth, but rather that animals have their young age in order to play.

Groos believed that the behaviour of mammals, and of people in particular, is so complex that they need a period of youth in which their preparation for adulthood culminates through games. He considers games to be a form of instinctive behaviour which builds foundations for the development of intelligence and, similarly to the imitation instinct, it substitutes other, more primitive instincts. Groos'

opinions that games are a means of general development were widely accepted, and most are acknowledged as truthful even today.

M. Montessori (1870–1952), the important Italian pedagogue, used the works of F. W. A. Fröbel, J. H. Pestalozzi and J. J. Rousseau when building her theory of education. She elaborated a supporting program to help pupils with writing and reading disorders, and she designed a system of education and upbringing for children aged 3–6. She laid importance on creating a stimulating environment for children which provides motivation for pupils' development. She thinks that one of the main teacher's tasks is to create and maintain such stimulating environment. In her education system, Montessori focused on overall child development: the development of their sensory-motor skills, vocabulary, preparation for writing, reading and simple mathematical operations, but also the overall development of pre-school pupils' behaviour. In order to develop these abilities and skills, Montessori widely used games as a natural form of activity for children. She supported the opinions of K. Groos about the major role of games for the development of young people. (**Britton, 1992; Röhrs, 1994**)

J. Piaget (1896–1980) and **B. Inhelder** (1913–1997), renowned French psychologists, carried out comprehensive studies of the functions of games in children's lives. According to their research, there are four basic categories of games. The original and most simple form of a game is an

"*exercise game*". In this, people apply certain abilities only out of "functional joy" or out of joy to cause something and to apply their newly-acquired knowledge. (Such behaviour can also be seen with adults, e.g. when trying a new car, computer, etc.) After this game stage there is a *symbolic game*, culminating between the ages 2–3 and 5–6. The basis of this is a playful transformation of reality that children assimilate into their needs. Such assimilation is based on a symbolic language which the child forms and can change creatively when necessary. Thanks to the symbolic game, children can act with no pressure and sanctions within conditions they adjust themselves, contrary to reality where they adapt to given conditions. The symbolic game therefore has great significance for both the emotional and intellectual balance of children and their development in these two areas. An example of a symbolic game is playing Indians, playing school... The third stage in the development of young people is *games with rules* (marbles, hide-and-seek, tic-tac-toe etc.). In these games children learn from each other (or an adult might be present). They are mainly important for children's socialization, the development of their social lives, and the development of their ability to cooperate and act on the basis of certain rules. The fourth type of game is *constructive games* which represent a transition between the symbolic game and activities which already have the nature of 'serious' work. In these games symbols are gradually objectivized, and reality is actually adapted (mechanical constructions, solving

various problems and puzzles, intelligent creative activity). (**Piaget** and **Inhelder**, **1997**)

We can see from the above-mentioned that both Piaget and Inhelder acknowledged the general function of games for children's development. According to their research, games develop sensory-motor skills, emotional and intellectual areas, as well as the imagination. The importance of a game was also confirmed in terms of children's socializing, the development of their ability to cooperate, as well as preparation for constructive creative activity, and the ability to solve the problems necessary in adult life. Therefore, on the basis of the results of their work, games are a necessary and important part of education and upbringing.

L. S. Vygotsky (1896–1934), the Russian psychologist, studied children's cognitive development in detail. He mainly studied the relation between language and thinking. In his opinion, cognitive abilities and thinking patterns are not primarily determined by inborn factors, but are a product of activities performed within the social institutions of the culture where the person is growing up. A game, as an activity, can create a suitable condition for the development of cognitive and thinking skills. (**Rozycki** and **Goldfarb, 2000**). In his theory, Vygotsky emphasizes the socializing role of games.

J. S. Bruner (1915), an American psychologist, significantly contributed to the evolution of cognitive psychology. He dealt with an efficient manner of education

and a suitable arrangement of the curriculum on the basis of the theory of constructivism. The basic idea of Bruner's theory is the opinion that learning is an active process in which pupils construct new notions and concepts on the basis of the knowledge and experience known to them. The teacher's task is to motivate pupils to learn new knowledge, while the teacher has to transform the knowledge to be learned in such a way that it is appropriate to the pupils' comprehension skills. Here Bruner was interested in the role of games in education, which he found equally as important as J. Piaget and B. Inhelder. In his research, e.g. he emphasized the ability of children to concentrate on an activity during a game, on the means of playful behaviour, and on obtaining information. In this way, a 'serious' activity is usually focused on its results. Therefore Bruner considers games to be a suitable educational method. For example, in the first steps of schooling, he suggested developing pupils' logical thinking through constructive games (**Bruner, 1960, p. 46**).

The evolution of the Reform Pedagogy in the late 19th century and 20th century was crucial for the use of games in teaching. An active, creative and motivating form of education was coming to the foreground. Certain new educational theories started to consider games as one of the main teaching methods.

J. Dewey (1859–1952), the founder of Pragmatism, emphasized the natural cognitive and educational function of games. Therefore he considered a game to be an essential

method of education. A game and its nature meet Dewey's requirement: "*To join school with life, to turn it into a place for children where they learn directly by life, instead of being only a classroom where tasks are assigned.*" (**Stojan** and **Jůva, 1995**). Dewey places great importance on the educational role of games. In his work (**Dewey, 2010, 2011**) he says that when bringing up children, all nations have largely relied on games and playful activity, as these activities teach children about the world they live in; children discover in them a lot about the activities and processes necessary for their lives.

R. Steiner (1861–1925), the Austrian scientist and philosopher, elaborated a method of education used at Waldorf schools. He emphasised the overall development of pupils, their knowledge, thinking, social skills, as well as their will power and spiritual values. He thinks that education should consider current children's needs which change with their physical, mental and emotional development. He requires that the main activities of children under 7 are games, drawing, learning about nature and common-life objects. In his theory, a game is considered to be a necessary condition of the comprehensive development of the human character. Steiner's current followers also emphasize the role of games in education (**Jenkinson, 2001**).

The French Group of New Education – GFEN (*Groupe Français d'éducation nouvelle*) attributes a significant educational role to games. This group of French pedagogues and didactic specialists both theoretically and practically try

to improve school education and upbringing. It emphasises teaching procedures motivating pupils to be active. As one of the most important members of this group, **H. Bassis** says that didactic games have a major place in their educational concept (**Štech** and **Bassis**, **1991**, **p. 25**):

From the pedagogic point of view, our didactic approach is based on the importance of games, i.e. on our finding that each game, in addition to stimulating activity and pleasure, has many specific qualities in its nature. It has a socializing effect, it inexorably forces the respect of limitations given by rules (exclusion from a game represents a sanction), it evokes an unusual willingness to make great effort and to go beyond one's limits, i.e. it has qualities which all moralizing expressions related to work dream about as an unachievable target.

As we have seen in this chapter, the importance of games for education is an idea which has been developing since ancient times, and it is supported by the opinions of numerous renowned and respected professionals.

3 MATHEMATICS TEACHING RESEARCH WITH DIDACTIC GAMES

The art of teaching is the art of assisting discovery.

M. Van Doren, (1943)

Developments related to didactic games as an integral part of education is still ongoing. A wide group of teachers use didactic games increasingly in their everyday work. Pedagogues and didactic experts examine didactic games both theoretically and practically as a method of mathematics teaching. This chapter is going to deal with the results of some research on the use of didactic games in mathematics teaching.

B. Onslow (1990) examined the positive effects of social interaction among pupils during didactic games. He continued with the work of **G. Bright**, **J. Harvey** and **M. Wheeler** (1985). He found that discussions among pupils or between pupils and the teacher are necessary during games in order to clarify conceptual conflicts and to introduce new terms and mathematical processes. The author set the following requirements which need to be fulfilled if the use of didactic

games has to increase the efficiency of mathematics education:

- Didactic games must be integrated in the math curriculum in an appropriate way, using consistent language, materials and symbols.
- The participation of pupils must be active throughout the whole game.
- Suitable interventions of the teacher are important because of targeted game management in order to help pupils acquire new terms, mathematical processes and ways of thinking.

J. Randel, B. Morris, C. Wetzel and **B. Whitehill** (1992) state that the use of didactic games in mathematics teaching may be beneficial to motivate pupils to work during the lesson, and to improve their lesson performances. According to the mentioned authors, a better understanding and remembering of the subject matter through games is conditioned by the active involvement of pupils in the games.

The role of didactic games in teaching has also been researched from many theoretical and practical aspects. **S. Pulos** and **C. Sneider** (1994) found that a suitably selected didactic game with a correct structure and focus help pupils acquire new mathematical terms and skills. These researchers recommend involving didactic games in the mathematics curriculum as an alternative activity. They discovered that the experience acquired through a suitable didactic game, used after pupils are familiar with certain

mathematical terms and skills, help pupils to understand them better and to remember them for longer.

L. Steffe and **H. Wiegel** (1994) focused on the ways didactic games can help pupils in constructing their mathematical reality, and improve their motivation and interest in mathematics. Didactic games brought about a spontaneous activity at math lessons, and the development of the mathematical thinking of pupils. Social interaction among pupils was also seen positively.

The Australian university pedagogue **G. Booker** deals with the active use of games in mathematics teaching at elementary schools. In his work *The Maths Games* (2000) he describes his experience and observations from didactic games. Let's mention a few:

Pupils find games a fun activity which not only motivates them, but also completely captures them, which is necessary for constructive teaching. Children who are not willing to learn to please their parents or a teacher, or because they have been told that maths will be necessary for their future life, are often willing to learn in social interaction with other pupils. Games offer a real context to pupils. …

For these reasons, games have an important place in mathematics education. They offer conditions where mathematical concepts may be constructed and developed. Games improve pupils' problem-solving skills through their need to discover and use new strategies, and they generally

improve other pupils' skills through their use in a motivating activity. They support social interactions that lead to learning.

The staffs of **The Mathematics Assistance Centre** at Griffith University, Brisbane, Australia, have carried out several research projects focused on didactic games. Such research was related to the use of didactic games as an integral part of teaching, enabling the acquisition of new knowledge, and the development of an understanding of new mathematical notions (**Booker, 2000**). Their aim was to find a mechanism to implement a constructive attitude to mathematics teaching. In the constructive attitude, pupils learn on the basis of their own experience through suitably selected activities. Didactic games are one such activity. The games also served to develop the language necessary to work with mathematical concepts. Let's try to briefly summarize the results of the research:

Didactic games:

- offer a real context to pupils where they can become fully involved, and that supports constructive teaching;
- increase the subjective value of mathematical knowledge for pupils, as this knowledge is necessary to participate in the game which is a desired activity;
- help pupils construct mathematical concepts through manipulation of objects as a part of the game, verbalization of their activities, thoughts and attitudes;
- require pupils to respect the rules of the game, and that is beneficial for math disciplines based on rules;

- are more efficient, when constructed on the basis of mathematical ideas, and understanding of certain mathematical notions or mastering certain mathematical skills is necessary for the game;
- support pupils to creatively construct new ideas which they have to defend in front of other players;
- they give stimuli to check and verify the mathematical processes of other players, in this verification pupils rely more on their own verification of correctness of these processes, instead of relying on the statements of an external authority (teacher, text book, etc.);
- improve the self-esteem and self-consciousness of pupils, as accidental game elements enable each pupil to win;
- enable the teacher to focus on the assessment of a true picture of pupils' abilities, instead of assessing their results in artificial conditions.

Many Slovak and Czech pedagogues also have much experience using didactic games during math lessons. Now we will present some of their opinions about the role of games in teaching, conditions and forms of their use, and practical observations from integrating games in the educational process.

In her works, **V. Kárová** (1994, 1996) formulated the requirement to use didactic games as a full-value teaching method. According to the author, games form the following pupils' qualities necessary for efficient education:

- overall positive relation to school and to the educational process;
- inner motivation to improve their knowledge, skills and abilities;
- self-assessment and self-control of their own activity.

E. Krejčová and **M. Volfová** (1994) highly value the role of a game as a vital part of education. According to the authors, using didactic games in teaching increases the interest of pupils in active work during math lessons and mathematics in general, and generally improves the course of lessons. The authors also find it positive that pupils must integrate the knowledge from different areas of mathematics, but also from different school subjects. On the basis of their experience with the use of didactic games in teaching math, the authors formulated the following requirements for the suitable integration of games in education:

- Games should be attractive and catchy for pupils.
- Games should correspond to age-related peculiarities and individual children's abilities. Younger pupils mainly like games full of elements of mystery and obscurity, pupils weaker in mathematics prefer team games, gifted and older pupils like individual games.
- Each game has to have clear and understandable rules which are carefully followed. Sanctions have to be determined for the potential breaking of rules (e.g. bad points). It is not appropriate to change rules for no purpose.

- Good organization and material equipment for the game is a must.
- It is not appropriate to introduce new games too often.
- We should never use games in teaching accidentally. Each game has to serve a certain didactic target which we want to achieve in teaching math.
- We should try to motivate as many pupils as possible with the game, ideally the entire class simultaneously. Each pupil should have the chance to be successful in the game, whether individually or as part of a team. In order to differentiate pupils depending on their abilities, it is appropriate to prepare less or more demanding variants of the given game.
- When selecting a didactic game, we prefer a game which will engage as many senses in each pupil as possible, which develops their varied skills and knowledge.

A. Masariková (1994, pp. 37–38) also points out the potential of didactic games in her statement:

A didactic game can be a means of both teaching and learning, it uses the natural desire to know and play. It is a source of motivation, it helps to focus attention, it increases the thinking activity, forms a positive relation and interest in the contents of teaching. That is a precondition for understanding and remembering basic facts resulting from the contents, and that gives the joy of achievement.

A. Masariková in cooperation with **J. Ivanovičová** (1999) dealt with the positive aspects of using didactic games in education. The authors believe that a great benefit of didactic games is the active work of pupils with the subject matter they are dealing with at the lessons. This direct contact enables pupils to acquire more exact, systematic and durable knowledge. In the methodology of using didactic games, the authors emphasize the appropriateness of games for the age, abilities, knowledge and interest of pupils.

J. Cejpeková (1996) describes the positive effects of games on the development of pupils in classes 1–4 at elementary schools. The author sees the potential of didactic games in their following functions:

- they activate pupils' personalities;
- they develop memory, imagination, attention, thinking and speech;
- they develop and cultivate the emotions of pupils, and support emotional learning through experience;
- they deepen self-knowledge and strengthen self-confidence;
- they enable social learning, and prepare for different social situations;
- motivate, develop interests, satisfy needs, lead to independence and creativity;
- support sensory-motor learning;
- have a significant relaxation effect.

According to **Š. Kováčik** (1999), the main areas which the educational use of didactic games in math can be focused on are:

- discovering and revealing new connections;
- practicing and revising the subject matter;
- developing thinking and using knowledge.

M. Zelinová (1999) made a detailed analysis of the functions of games in the development of children's personalities. According to the author, games have a significant function in the development of the following character aspects:

Non-cognitive areas:

- emotional expressions and positive experience, increase of self-consciousness;
- increase in activity and motivation;
- social behaviour, improvement of social skills;
- love of positive values;
- creativity, joy of creative activity.

Cognitive areas:

- sensory-motor abilities;
- memory;
- evaluative thinking;
- creative thinking.

In his works, **P. Vankúš** (2006, 2007, and 2008) studied the influence of using didactic games in teaching math in classes 5–9 at elementary schools. The main output of the research was the author's finding that the integration of

didactic games has a positive effect on pupils' attitudes to mathematics and its teaching.

One of the most positive qualities of games is the fact that games are a natural way for children to learn. It is also important that children find games to be a wanted activity that they like. Games have a close context for children, and thus they help them overcome some didactic obstacles in the teaching process of mathematics. Let's describe the mentioned obstacles in more detail:

Didactic obstacles are caused by selecting educational methods and elaborating the teaching content (e.g. formal teaching of combinatorics through drilling formulae, without really understanding them). These obstacles may be overcome by a suitable choice of teaching methods and the curriculum. (**Spagnolo, 1998**)

Considering the data stated in this part of the work, didactic games may help pupils overcome didactic obstacles in the teaching process of mathematics. As we have said, a suitable didactic game is an activity appropriate to the age, abilities and interests of pupils. In addition, games are a natural way for children's education. These reasons indicate the didactic game's potential as a method leading to the elimination of didactic obstacles.

To conclude, we will present a quotation from the work of **J. Cejpeková** (1996, p. 23):

From the point of view of the basic attribute of the method, we may understand a game as a teaching method. In addition

to stirring action, it also fulfils the basic functions of a method, namely the educational, pedagogic and developing functions. It helps pupils not only acquire knowledge, but their moral and personal qualities are formed as well. With regular application of games, a favourable atmosphere is created during lessons because children like games. Games help them to learn without stress. Along with learning, they feel joy, have fun, and have a pleasant emotional experience not only from the game but also from teaching. These emotions and experience in the teaching process may be much more important and significant for children's lives than one-sided mental recognition. A great benefit of this method is also the fact that it does not require any complex secondary motivation. And lastly, it is valuable that children come to school, where they play, with expectations and joy every day. There is hardly any teaching method with such convincing preconditions to have both an educational and upbringing effect on children as a game.

Researchers dealing with didactic games in mathematics education agree that games have many positive qualities, and may be a beneficial method for pupils' work during mathematics lessons.

4 SELECTION OF SUITABLE DIDACTIC GAMES AND THEIR METHODOLOGY OF USE

Let's find a way for teachers to
teach less and for pupils to
understand more.

J. A. Comenius, (1954)

On the basis of our experience with didactic games, as well as the experience of professionals in this area, in this chapter we will formulate some recommendations for the practical use of didactic games as part of mathematics teaching.

When using didactic games during mathematics lessons, the right **choice of a didactic game** is important, as well as a suitable **work methodology** with this game.

Even before using a didactic game in the teaching process, it is necessary to analyze several factors. It is necessary to determine the **educational goal** we want to achieve. On the basis of the educational goal, we will choose the suitable didactic game. When selecting a game, we must consider its

appropriateness for particular pupils, depending on their age, maturity and interests. We will also think about possible **variants** of the given game in order to differentiate its difficulty.

The choice of game is also influenced by its **organizational and performance difficulty**. This difficulty is given by the demands of the teacher as the organizer and controller of the course of the game, as well as by the difficulty of actions expected from pupils. **Material equipment** is also part of the game's performance. In each didactic game, it is necessary to consider its other characteristic features. In particular, we are looking for answers to these questions:

- How many pupils will the didactic game involve and on what cognitive process level?
- What knowledge, abilities, skills and character features does it develop?
- What is the expected effect of the given didactic game on the efficiency of the teaching process?
- What effect does it have on the attitude of pupils to the teaching process and the subject?
- What are the motivational effects of the given game on pupils?

On the basis of the above-stated, we can create an 'ideal didactic game' model: **An ideal didactic game is suitable to achieve the determined educational goal, appropriate, and interesting for pupils. As for the organizational and performance difficulty and material equipment, it is**

rather simple, but still an attractive game with its course and content. This game engages the whole class at the same time, while it gives preference to the creative activity of pupils. An ideal game develops a wide range of knowledge, abilities, skills and positive character features of pupils. Its integration in the teaching process will result in increased efficiency of the teaching process, compared with teaching without this game. It has a positive effect on pupils' attitudes to mathematics, and motivates them to further education and development.

When selecting a didactic game, we are trying to come as close as possible to the 'ideal didactic game' model.

Now we will speak about the methodology of using didactic games during mathematics lessons.

We usually present didactic games during the lesson in which we want to use them, so that pupils remember the rules of the game. If the game is not the only activity performed during the lesson, we usually do it at the end. Firstly, not to have to rush the game and to have time for other activities. Secondly, due to the fact that a game, as a relaxing activity with positive effects on pupils, is a suitable conclusion for math lessons.

At the start of the didactic game presentation, we will tell the pupils its name, which should be appropriate and also attractive for pupils. Then we will tell pupils the rules of the game. The explanation of rules should be demonstrative, preferably with particular examples of game situations. It has

been proven that it is appropriate to perform a demo game or part of the game activity after presenting the rules, where pupils can check if they have understood the rules correctly.

Then the didactic game itself is carried out. If necessary for the game, we split pupils into teams. Mostly there are 5 or 6 team members. Ideally, teams should be equal both in the number of pupils and in mathematical knowledge and skills. When using didactic games on a long-term basis, we can create a fixed structure of teams in line with this requirement, also taking into consideration relations among pupils and their preferences of who they want to team-up with.

The task of the teacher during didactic games is usually to check if the rules are being followed, or to control the course of the game from the organizational point of view. If pupils intentionally repeatedly break the rules, they need to be disciplined – depending on the situation and conditions, e.g. by a worse assessment in the given game. After the end of the game, it is necessary to evaluate its course and the work of individual players. Not only should pupils' results be considered in this evaluation, but also their effort. Eventually, each pupil who has actively worked during the game should be rewarded and encouraged. It helps to increase pupils' motivation to participate in game activities. Secondarily, this motivation may be transferred to other educational activities performed during math lessons.

Awarding points has proven effective in our practice, with more successful players obtaining more points and less successful players fewer, but not 0 points. Scores of more games are counted. For a certain number of points determined in advance, pupils get an A for activity during math lessons; or e.g. 5 pupils with the highest overall number of points will get an A after the end of one thematic unit in which we used the given didactic games. The advantage of this assessment is motivation on the one hand, and the elimination of the negative effects of assessments (each player will get a non-zero number of points, there is no threat of a bad grade). Applying such point-based assessment is also suitable when using an overall point-based assessment of pupils in their math lessons, while we will add the score for didactic games as bonus points to the total number of points awarded to the given pupil for other activities.

We use didactic games in teaching only when we are convinced of the benefits of the given game in comparison with teaching without the game. For example, instead of a routine practice or revision of some subject matter, we may achieve this goal in a more attractive way for pupils – through a suitable didactic game. It is not necessary to always use a new type of didactic game. Using a known game has the advantage that pupils know the rules of the game and its organization. So they can concentrate on the game activity itself. You also save the time that you would

otherwise spend explaining a new game to the pupils, which is also an advantage. It is necessary to vary the games however, to make lessons more interesting.

Teacher's preparation before the first use of the game can be rather demanding. But to comfort teachers, when you repeatedly use the given didactic game with the same content, you can use the same material aids. Then we can use games without considerably higher, sometimes lower, demands on preparation, in comparison with teaching without the game. A really great advantage of didactic games is active, independent pupils' work. Math lessons with integrated didactic games pose lower demands on teachers, in comparison with no-game classes, in terms of encouraging pupils to work and maintaining discipline.

5 COLLECTION OF DIDACTIC GAMES

The only teacher worthy of this
name is one who inspires the spirit
of free thinking and develops
a sense of personal responsibility.

J. A. Comenius, (1954)

At the start of this chapter, we have good news. Having read the steps to select a suitable didactic game in the previous chapter, many must have thought that it is quite difficult to choose the right game (and we agree). This is why we have included the Collection of Didactic Games in this chapter, selected in such a way that they are suitable for use in particular thematic areas of mathematics at lower secondary school (pupils 10–15 years old).

The description of each didactic game presented here starts with its **name**. In the **thematic area of the game** we mention the math areas for which the given game is primarily determined. Then there are the **educational targets of the game**. Then we describe the **game environment**, particularly any aids necessary, or a division of pupils into teams, as well as the approximate duration of the game. Then the **game procedure** follows, i.e. the activities of the teacher and pupils

during the game. This procedure contains the rules of the game which control the course of the game activity. In a separate part we propose a **final assessment** of the pupils' work. Finally, we present some **benefits of the game** for the educational process.

Games in the Collection are divided into the following categories:

- **Cooperative games for teams**. This category includes games where a team of pupils cooperate. This category is further sub-divided into **games for multiple teams** and **games for teams of 2**.
- **Individual games**. This category includes games where each pupil plays on his/her own. We further sub-divide these games into **games for individuals** and **pair games**. Individual games are of a solitaire type, i.e. games where each pupil really plays as an individual and tries to achieve the goal set by the rules of the game on his/her own. Pair games are of the chess, draughts, etc. type, i.e. games where pupils play in pairs, they have opposite interests, and try to beat their play mate.

The games presented in this Collection derive from a variety of sources. In particular, we will mention the following works, ordered by year of publication, described in more detail in the Bibliography at the end of this publication:

M. Gardner *Mathematical puzzles & diversions* (1959),

M. Gardner *Martin Gardener's Sixth Book of Mathematical Games from Scientific American* (1971),

M. Gardner *Matematičeskije dosugi* (1972),

M. Hejný *Geometria naučila človeka myslieť* (1979),

F. Roth *Matematická hra ako prostriedok rozvoja kognitívnych funkcií* (1980),

V. Žmuráň *Matematické hry* (1985),

V. Burjan, K. Bachratá and **H. Bachratý** *Odborný program matematických krúžkov na 2. stupni základnej školy* (1989),

V. Burjan and **Ľ. Burjanová** *Matematické hry (1991),*

T. Houška *Škola hrou* (1991),

J. Melichar and **M. Červenka** *Matematika hrou* (1993),

V. Kárová *155 her ve vyučování matematice a ve školní družině na 1. stupni ZŠ 1. a 2. část* (1994),

D. Môťovská *Netradičné metódy vyučovania matematiky na základnej škole a v nižších triedach osemročných gymnázií* (1994),

E. Krejčová and **M. Volfová** *Didaktické hry v matematice* (1994),

J. Brincková *Didaktická hra v geometrii* (1994),

L. Horník *Matematika hrou* (1994),

V. Kárová *Didaktické hry ve vyučování matematice v 1.–4. ročníku základní a obecné školy* (1996),

J. Cejpeková *Hra vo vyučovaní na prvom stupni základnej školy* (1996),

K. Foltínová and **J. Novotná** *Matematické hry a soutěže na druhém stupni základní školy* (1997),

Š. Kováčik *Didaktická hra – spestrenie hodiny matematiky* (1999),

D. Ivančíková *Hry ako súčasť vyučovania matematiky. Hry odbúravajúce stres* (2002),

P. Vankúš *Hry ako súčasť vyučovania matematiky* (2002).

Many of the games appear in several of these works, in different modifications. When preparing the Collection, we have also adjusted some of the games to our requirements. The origin of some didactic games is uncertain, and it is difficult to find their original author. Therefore, in our Collection we do not mention the particular sources of games from which we obtained the game ideas.

In the following part of the publication, you can find 30 didactic games giving you ideas for some game activities during math lessons. Naturally, the mentioned games can be modified depending on the situation, or they can be adjusted to suit other areas of mathematics.

COOPERATIVE GAMES

Games for multiple teams

1 Bingo

Thematic area:

This game is suitable for all thematic areas in mathematics.

Educational targets:

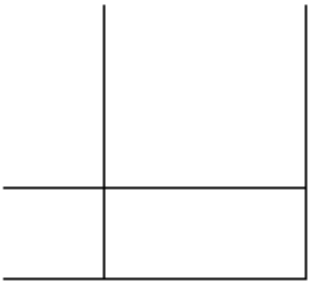
To practice solving various mathematical tasks, feedback on mastering the subject matter. The game also develops pupils' cooperation.

Game environment:

Pupils and the teacher: Teams of 5–6 pupils cooperate. The teacher has an organizational and controlling role.

Material environment: Set of tasks for each team, a ticket (paper with blank boxes, see *Picture 5.1*).

Game duration: 40 min



Pic. 5.1 Ticket for the *Bingo* game

Game procedure:

First we distribute task assignments and the result sheet (ticket) to all teams. The assignment contains a certain number of tasks, while their number and difficulty must enable the pupils to solve all tasks in the course of the game. The tasks are numbered in sequence, the number of tasks is higher than the number of boxes on the result sheet. Another (secret) number is also assigned to each task – it is a sequential number of the given task which we get by the random shuffling of tasks (*Demonstration 1*). For example if the assignment contains 16 tasks, the ticket has 9 blank boxes. The pupils will randomly write 9 non-negative numbers from 1 to 16 on their tickets, and they will solve the tasks from the assignment. They consult the task results with the teacher. If they have the correct result, the teacher will show them the secret number corresponding to the given task. If this number is written on the ticket of the given team of pupils, the teacher will mark it.

The aim of the game is to mark all boxes on the ticket, i.e. to achieve 'bingo'.

Identification of the tasks with secret numbers gives the game an element of surprise and chance. If pupils are lucky when filling in the ticket, they will have bingo even if they have not solved all the assignment tasks. It means that also weaker pupils have the chance to achieve better results and experience success.

Final assessment:

All pupils in a team will get a certain number of points for the activity for each correctly solved task (depending on the difficulty of tasks). Pupils will get bonus points for 'bingo' (points should be sufficiently high to motivate and please pupils).

Benefits of the game:

Active work of the whole class, inner motivation of pupils thanks to the game's attractiveness. Development of the ability to cooperate.

Demonstration 1 Secret numbers of single tasks in the *Bingo* game

	Secret number
Task n. 1	7
Task n. 2	4
Task n. 3	11
Task n. 4	5
Task n. 5	6
Task n. 6	9
Task n. 7	13
Task n. 8	8
Task n. 9	16
Task n. 10	10
Task n. 11	1
Task n. 12	3
Task n. 13	2
Task n. 14	14
Task n. 15	15
Task n. 16	12

2 Circles

Thematic area:

This game is suitable for all thematic areas in mathematics.

Educational targets:

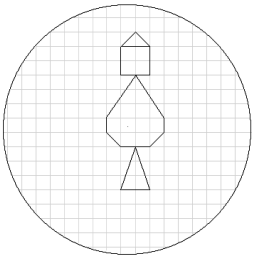
To practice solving various mathematical tasks, feedback on mastering the subject matter.

Game environment:

Pupils and the teacher: The class is divided into teams of 5–6 pupils. The teacher has an organizational and controlling role.

Material environment: Task cards with a corresponding number of points (see *Picture 5.2* and *Demonstration 2*) for each team. The cards can have a circular shape – hence the game's name.

Game duration: 15–30 min



Pic. 5.2 Game card for the *Circles* game, the task is to determine the area of geometric shapes shown on the cards. It is a thematic unit of *the Area of Geometric Shapes*.

Demonstration 2 Tasks for the *Circles* game for the thematic unit dealing with the *Solution of Algebraic Expressions*.

10-point tasks:

1) $(x - y) + (y + z) - (y - z) + (y - x) =$

2) $(3x - 7y) + (-4x + y) - (x - 5y) =$

3) $(6m^2 - 3m) : 3m =$ $(m \neq 0)$

4) $(-56ab + 24bc) : (-8b) =$ $(b \neq 0)$

5) $(0.5a^2 - 0.5) : 0.25 =$

6) Determine the value of $x + xy - y$ for $x = 5$ and $y = -0.4$

5-point tasks:

1) $2(y - 1) + (y - 2) =$

2) $0.5(2p - 4) + 0.4(5p - 10) =$

3) $(-2y) \cdot (-9y) =$

4) $(4.5r - 18) : 0.9 =$

5) $(-3c - 9d) : 0.3 =$

6) Determine the value of $(x + y) \cdot (x - y)$ for $x = 3$ and $y = -2$

Game procedure:

The teacher divides the cards into two groups, depending on difficulty (cards of different difficulty have different colours). Pupils will solve the tasks on the cards. The pupils choose the difficulty themselves by selecting a task from a particular group of cards. If they succeed in solving the task, their team wins the corresponding number of points. The teacher assesses the correctness of the task solutions, and records the scores of single teams. Tasks are numbered to enable faster control, the teacher has a result for each

task. Pupils put down the solutions on a special piece of paper, after a certain time (15–30 min) the results are checked. The goal of the teams is to obtain the maximum number of points possible.

Final assessment:

After the end of the game, the scores of single teams are counted, and a ranking of teams is made on the basis of the obtained points. Pupils are rewarded for activity with points – pupils of the winning team obtain the highest number of points, pupils of the team ranked last obtain the lowest, but not zero points.

Benefits of the game:

The differentiated difficulty of tasks, inner motivation of pupils through competitiveness among teams, the opportunity also for weaker students to contribute to the team's success, active work of the whole class, work of pupils in an attractive context, feedback on the knowledge level of pupils in single teams. Pupils' social skills develop through their interaction in teams.

3 Puzzle

Thematic area:

This game can be used for solving various mathematical tasks whose solution is an explicit number.

Educational targets:

To practice solving various mathematical tasks, feedback on mastering the subject matter.

Game environment:

Pupils and the teacher: The class is divided into teams of 5–6 pupils. The teacher has an organizational and controlling role.

Material environment: Numbered assignments of tasks, each task is on a separate piece of paper (*Demonstration 3*). A picture cut into the number of pieces equal to the number of tasks. Each part is numbered with non-negative numbers, starting from 1, on the side without the picture. There can be anything in the picture, e.g. an animal, a general object, a famous person... Magnetic board or a notice board.

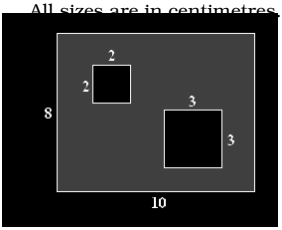
Game duration: 20–40 min

Demonstration 3 Task for the *Puzzle* game for the thematic unit of *the Area of Geometric Shapes*.

Task 7

Count the area of the dark zone in the picture.

All sizes are in centimetres.



Game procedure:

Place the picture cut into pieces on the magnetic board or notice board so that the pupils can see the numbered side. Pupils of each team select one of the given numbers, corresponding to the part of the picture they want to turn over (each team a different number). On the basis of that number, the teacher will assign a task with the same number to the pupils. After the task is solved, the teacher will check its correctness. If the result is correct, he/she will turn over the corresponding part of the picture so that the part with the picture is visible. The pupils of the team which has solved the task have the opportunity to guess what the picture is depicting. If the result was incorrect, the stated part of the picture remains unturned until the end of the game. After each team has solved the first task, we continue with the next task, until all the tasks have been solved.

Final assessment:

All pupils in a team will get a point for the activity for each correctly solved task. If they guess right what is depicted in the picture, the pupils of the given team get bonus points for the activity.

Benefits of the game:

Inner motivation of pupils through competitiveness among teams, opportunity also for weaker pupils to contribute to the team's success, e.g. by guessing what the picture depicts. Active work of the whole class, work of pupils in an attractive

context, feedback on the knowledge level of pupils, pupils' social skills develop through their interaction in teams.

4 Builders

Thematic area:

This game is suitable for the thematic areas of *Perimeter of Geometric Shapes*, *Area of Geometric Shapes* and the *Volume and Area of Geometric Shapes*.

Educational targets:

To practice converting the units of length, area and volume. To train the calculation of area and volume. Development of spatial imagination and skill in geometric constructions.

Game environment:

Pupils and the teacher: The class is divided into teams of 5–6 pupils. The game is played on an asphalt playground or the classroom. The teacher has an organizational and controlling role.

Material environment: Construction plan for each team (*Demonstration 4*). Chalk suitable for drawing on asphalt. A ruler for each team.

Game duration: 40 min, the game content can also be used for more math lessons.

Game procedure:

The teacher prepares a simple construction plan for each team. It can be a plan of a house, maze, park, etc. Sizes in the plan are written in different units of length. The task of

the teams is to 'construct' a building true-to-scale 1 : 10 on the asphalt playground with chalk. When the construction is ready, the teacher approves it and gives the pupils tasks, e.g. to count the areas of single construction parts, the volume of the construction with the height of walls given, the cost of painting the construction, the option to change the sizes of the construction and so lower the costs, etc. Such accompanying tasks can also be solved at the next math lesson.

The stated game can also be played in the classroom. Pupils draw the construction on the blackboard with chalk, or with their drawing tools on paper. In this variant it is necessary to consider the number of teams and suitable 'construction' sizes when creating the construction plans.

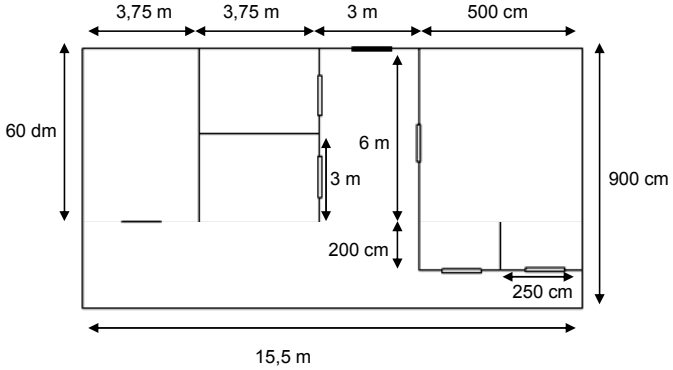
Final assessment:

On the basis of the correctness of the 'construction' and the solution of the assigned tasks, each team member will get a certain number of points for the activity.

Benefits of the game:

Active work of pupils in a motivating and natural context. Development of various pupils' skills. Building skills of team cooperation.

Demonstration 4 Construction plan for the *Builders* game



5 Lines Contest

Thematic area:

The game is suitable for practicing solving tasks focused on various math areas.

Educational targets:

To practice task solving. Feedback on mastering the subject matter both for pupils and the teacher. The game develops pupils' ability to check the correctness of math tasks solutions and will result in the decrease in error rate.

Game environment:

Pupils and the teacher: The class is divided into 3 teams of pupils (you can use the natural seating of pupils in lines). The teacher has an organizational and controlling role.

Material environment: Set of mathematical tasks. They are arbitrary tasks which can be solved on the blackboard.

Game duration: 40 min

Game procedure:

Pupils of single teams take turns in solving tasks on the blackboard. The tasks are assigned by the teacher. While one pupil is solving the task at the blackboard, other pupils are counting at their desks and checking his/her solution process. When the pupil at the blackboard finishes, pupils of other teams assess the correctness of the solution.

If the solution has been unanimously *found correct* and *is indeed correct*, the team of the pupil who has solved the task will get a point. If the solution *has been said to be incorrect* and *there really is a mistake*, the assessing pupils will correct the mistake and the team of the pupil who has been solving the task will not get a point. If the solution *has been said to be incorrect* and *actually is correct*, and the pupil who has solved the task or his/her team-mates defend the correctness of the solution, this team will get 2 points. If the solution *has been said to be correct* and *actually is incorrect*, the teacher will advise the pupils thereof and correct the mistakes. The team of the pupil who has been solving the task will get 1 point in this case. This way of awarding points makes the pupils of all teams pay attention to the work of the pupil at the blackboard. The task of the teams is to obtain the maximum number of points possible.

Final assessment:

All pupils in teams will get points for the activity on the basis of the number of points achieved by their team in the course of the game.

Benefits of the game:

Active work of the class, inner motivation of pupils through competitiveness. Development of the ability to check the correctness of math tasks solutions, the pupils will notice the most frequently occurring mistakes through the game.

6 Chairs Contest

Thematic area:

This game can be used for mathematical tasks the solution of which is not time consuming and the result of which is explicit number.

Educational targets:

To practice solving various mathematical tasks, feedback on mastering the subject matter.

Game environment:

Pupils and the teacher: The class is divided into 3 teams. The teacher has an organizational and controlling role.

Material environment: Prepared assignments of tasks for the teacher. Pack of cards with task results for each team. (*Demonstration 5*)

Game duration: 20–40 min

Game procedure:

At the start of the game the teacher will give the same pack of cards to each team. Each pupil in the team will get the number of cards given by the number of pupils in the team and the number of tasks which the teacher has prepared. The data on the cards are the results of these tasks. The teacher

will read or write on the blackboard the wording of the first task. All pupils solve the task on their own. Having solved the task, all students who have the card with the result of the task then run to the blackboard. The aim is to get there sooner than the pupils of other teams. In order to determine their order more easily, and to enliven the game, 3 chairs are in front of the blackboard. The fastest pupil will sit down on the first chair, the second on the next and the last pupil on the third. Having checked the tasks and determined the pupils' order, the teacher will assign a next task. This process repeats until all the tasks have been solved.

Final assessment:

All pupils of the team will get a certain number of points for each pupil who has presented his/her card with a task solution correctly. The number of points depends on the difficulty of the tasks and the order of pupils of single teams. For example the team of the pupil who has presented his/her card with a correct solution of the given task first, will get 3 points. The team of the pupil who came second will get 2 points, the team of the third pupil will get 1 point. If a pupil comes to the blackboard with an incorrect card, his/her team will get 0 points.

Benefits of the game:

Inner motivation of pupils through competitiveness among teams. Active work of the class. Feedback on the pupils' knowledge level.

Demonstration 5 Wording of the tasks and a pack of cards with their results for the *Chairs Contest* game related to the subject matter of *Percentage.*

1) Task assignments

1. 10 % of 150
2. 25 % of 40
3. 50 % of 12
4. 75 % of 4
5. 30 % of 60
6. 70 % of 20
7. 85 % of 120
8. 50 % of 0.5
9. 120 % of 2
10. 100 % is 10. How much % is 2.5?
11. 100 % is 12. How much % is 9?
12. 100 % is 50. How much % is 10?
13. 100 % is 60. How much % is 20?
14. 100 % is 25. How much % is 35?
15. If 50 % is 50. How much is 100 %?
16. If 10 % is 0.5. How much is 100 %?
17. If 75 % is 9. How much is 100 %?
18. If 125 % is 20. How much is 100 %?

2) Pack of cards with the results of the given tasks

15	10	6	3	18	14
102	0,25	2,4	25	75	20
$33,\overline{3}$	140	100	5	12	16

7 Cipher Game

Thematic area:

The game is suitable for practicing solving simple tasks focused on various areas of mathematics.

Educational targets:

To practice task solving. Feedback on mastering the subject matter both for the pupils and the teacher. The game develops the pupils' ability to cooperate.

Game environment:

Pupils and the teacher: The class, pairs sitting at desks cooperate. The teacher has an organizational and controlling role.

Material environment: Set of mathematical tasks. They are simple tasks, their result has to be different for each task (see *Demonstration 6*). Blank key to decode the message and an enciphered text – it can be any text, e.g. a puzzle or a joke (see *Demonstration 7* – this enciphered text is a puzzle).

Game duration: 40 min

Game procedure:

At the start of the game, pupils are given a set of tasks and an enciphered message together with a blank key to decode it. There is one letter of the alphabet stated with each task in the set. The enciphered text consists of data, separated by commas. This data is the result of the given tasks. We will decode the text by exchanging the data for letters written next to the tasks which the corresponding data are the results for. Pairs of pupils solve the tasks and thus obtain the key to

decode the cipher. The aim is to decode the whole enciphered message.

Final assessment:

Each pair at a desk writes the results of single tasks in the key. After the end of the game, the teacher will check them. Pupils will get a certain number of points for the activity for each correct result. We can award bonus points for the correct decoding of the enciphered message. If the message is a puzzle, we can award bonus points also for guessing it.

Benefits of the game:

Active work of the class, inner motivation of pupils by the game's attractiveness and competitiveness. Development of the ability to cooperate.

Demonstration 6 Set of tasks used in the *Cipher Game* related to the thematic unit of *Area of geometric shapes*

Count the area of a rectangle with the following sizes. Write the result in units corresponding to the unit of the first stated figure.

a) 2.5dm and 20cm	A
b) 3m and 25 dm	B
c) 5.5cm and 40mm	C
d) 2.5cm and 50mm	D

Count the area of a square, if its side's length is:

a) 1m	E
b) 5dm	F
c) 7cm	G
d) 10mm	H

You know the area of a rectangle and the length of one of its sides. Determine the length of the adjacent side. Write the result in units corresponding to the unit of the first stated figure.

a) $3m^2$, 2m	I
b) $8cm^2$, 0.2dm	J
c) $6m^2$, 200cm	K
d) $50m^2$, 5m	L

You know the area of a square. Determine the length of its side.

a) $36mm^2$	M
b) $64m^2$	N
c) $81dm^2$	O
d) $9mm^2$	P
e) $144cm^2$	Q

You know the perimeter of a square. Determine its area.

a) 20cm	R
b) 12cm	S
c) 28dm	T
d) 24m	U

You know the area of a rectangle and the length of one of its sides. Determine its perimeter.

a) $30cm^2$, 10cm	V
b) $32cm^2$, 8cm	W
c) $50mm^2$, 10mm	X
d) $36dm^2$, 18dm	Y
e) $15dm^2$, 5dm	Z

Demonstration 7 Key and a ciphered puzzle for the *Cipher Game* related to the thematic unit of *Area of Geometric Shapes*

Decoding key:

A	B	C	D	E	F	G	H	I	J	K	L	M

N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Enciphered puzzle:

|| 9dm 8m 1m² || 9cm² 9dm 22cm² 3m ||

|| 12,5cm² 25cm² 1,5m 1m² 9cm² ||

|| 1,5m 8m || 25dm² 1,5m 26cm 1m² ||

|| 100mm² 9dm 36m² 25cm² 9cm² ||.

|| 100mm² 9dm 24cm|| 10m 9dm 8m 49cm² ||

|| 12,5cm² 9dm 1m² 9cm² || 1,5m 49dm² ||

|| 49dm² 5dm² 3m 1m² || 49dm² 9dm ||

|| 12,5cm² 25cm² 40dm || 49dm² 24cm 9dm ||

|| 9cm² 9dm 22cm² 3m 9cm² ||?

Answer to the puzzle:

8 Lotto

Thematic area:

This game is suitable for thematic units dealing with the *Arithmetic of Numbers and Expressions*, for the thematic unit of the *Area of Geometric Shapes*, for the thematic unit of *Percentage*, in the area of *Functions* as well as for thematic units dealing with *Solving Linear Equations and their Systems*. The stated game may also be used for solving various verbal tasks whose solution is a number or a numeric expression.

Educational targets:

To practice solving various mathematical tasks, feedback on mastering the subject matter. The game also develops the pupils' ability to independently assess their results, and their cooperation.

Game environment:

Pupils and the teacher: The class, pairs sitting at desks cooperate. The teacher has an organizational and controlling role.

Material environment: Set of tasks for each pair (see *Demonstration 8*), a ticket (sheet of paper with boxes where the result is written next to the number of the task; see *Picture 5.3*).

Game duration: 20 min

Demonstration 8 Set of tasks for the *Lotto* game for the thematic unit of *Area of Geometric Shapes*.

Task 1: We should paint a rectangular wall with the sizes 5 m by 3 m. Suggest how many cans of paint we should buy, if one can is sufficient to paint 5 m^2 of the wall.

Task 2: We are polishing a square table with each side 1.5 m long. One bottle of polish is sufficient for 2 m^2 of wood. How many bottles of polish do we need?

Task 3: In a flat is a rectangular room sized 4.5 m by 3 m, and another room in the shape of a square with each side 4 m long. How much larger is the area of the second room in comparison with the first?

Task 4: In a square room with each side 4 m long is a rectangular rug sized 3.5 m by 3 m. How many m^2 of the floor are not covered by the rug?

Task 5: A town square is 100 m by 50 m. How many people can stand on the square if you need 0.5 m^2 per person?

Task 6: We want to sow corn in a field. How many kg of corn will we need if the field is rectangular, sized 200 m by 50 m? We know that we need 0.5 kg of corn per 250 m^2.

Task 7: 30 g of fertilizer are used per 1 m^2. The gardener wants to fertilize 5 flowerbeds sized 2 m by 10 m. How much fertilizer will he need?

Task 8: A square land has an area of 400 m^2. Determine the length necessary to fence it. (clue: 20 x 20 = 400)

Task 9: The fence around a square-shaped playground has the length of 200 m. Determine the area of the playground.

n. 1	n. 2	n. 3
n. 4	n. 5	n. 6
n. 7	n. 8	n. 9

Pic. 5.3 Ticket for the *Lotto* game

Game procedure:

First we distribute task assignments and the result sheet (ticket) to pupils at each desk. Pupils solve the tasks and write their results on the ticket. Both the tasks and the ticket boxes are numbered. Each task result is written in the box with the same number as the task. After they finish working, the neighbouring pairs cross-check their results on the basis of the correct results which the teacher has prepared. They will cross any correct result, and they will leave all incorrect

or no results unmarked. The aim of the pairs is to get as many crosses on their tickets as possible.

Final assessment:

Both pupils at the desk will get a point for the activity for each correctly solved task.

Benefits of the game:

Active work of the class, inner motivation of pupils through competitiveness. The pupils' mutual cross-checking eases the load on the teacher. The game develops the ability to cooperate.

9 Relay

Thematic area:

This game can be used for mathematical tasks the solution of which is not time consuming and the result of which is explicit number.

Educational targets:

To develop the pupils' ability to carefully solve tasks in order to get the correct results. Feedback on the quality of knowledge both for the teacher and pupils.

Game environment:

Pupils and the teacher: The teams of pupils, 2–4 pupils in one team. The teacher has an organizational and controlling role.

Material environment: Each team of pupils will get a card with tasks (see *Demonstration 9* and *Demonstration 10*). They are tasks of the same type and difficulty.

Game duration: 5–10 min

Demonstration 9 Tasks for the *Relay* game in the subject matter area of *Linear equations*.

1) $9x + 12 = 36 + 3x$
2) $12\cdot\square + 21x = 90$
3) $13\cdot\square - 8x = 4 + 3x$
4) $x - \square + 2 = 2x + 4$
5) $2 + \square + 3x = 34$
6) $3 + x = -\square + 6x$
7) $3\cdot\square - x = 8x - 27$
8) $\square - 7x = 3x - 16$

Demonstration 10 Tasks for the *Relay* game in the subject matter area of *Percentage*.

1) 5 % of 240
2) □ % of 200
3) □ % is 48, determine 100 %
4) □ % of 29
5) □ % is 174, determine 100 %
6) □ % of 13

Game procedure:

The first pupil in each team gets a card with task assignments. This pupil will solve the first task, while he/she will write the result in the blank box in the assignment of the second task. Similarly, the next pupil in the team will check the result of the first pupil and solve the second task, while after he/she has done so, he/she will write his/her result in the assignment of the third task. In this way pupils continue with all the tasks on the card. Each subsequent result depends on the correctness of the previous task's solution. The aim is to solve correctly as many tasks as possible. After the end of the game, the teacher will check the correctness of solutions on the basis of the previously prepared results. For a quicker check, single cards and the tasks in them are numbered.

Final game assessment:

All pupils in a team will get a certain number of points for the activity for each correctly solved task (depending on the difficulty of tasks).

Benefits of the game:

This game makes pupils pay more attention and try to solve the tasks correctly, as the results of other team members depend on their solution. The game requires the active involvement of each player, though the game has a disadvantage – the players in the team are not active simultaneously, but one after another. That is why the tasks in this game must not require lengthy solutions. The game has a positive influence on motivation.

Games for Teams of 2

10 Cube with Letters

Thematic area:

The game is focused on the *Development of the Spatial Imagination of Pupils.*

Educational targets:

Practicing the subject matter dealing with the cube net in various forms. Development of pupils' spatial imagination.

Game environment:

Pupils and the teacher: Pairs at desks cooperate. The teacher has an organizational and controlling role.

Material environment: Assignment for each pair of pupils (see *Demonstration 11*).

Game duration: 5–10 min

Game procedure:

Pupils must mark which pictures represent the net of the given cube. The task has been made more difficult due to the need of a correct orientation of letters. The game can be organized as a contest, while the fastest pair to answer correctly will win. This game can also be played in a modified way – the game assignment will contain pictures of nets of various cubes, while some will be different nets of the same cube, and the pair of pupils must group all the nets corresponding to single cubes.

Final game assessment:

Each pupil of a pair will get a point for activity for every correctly marked net of the cube. For homework, pupils can construct cube models on the basis of the presented nets.

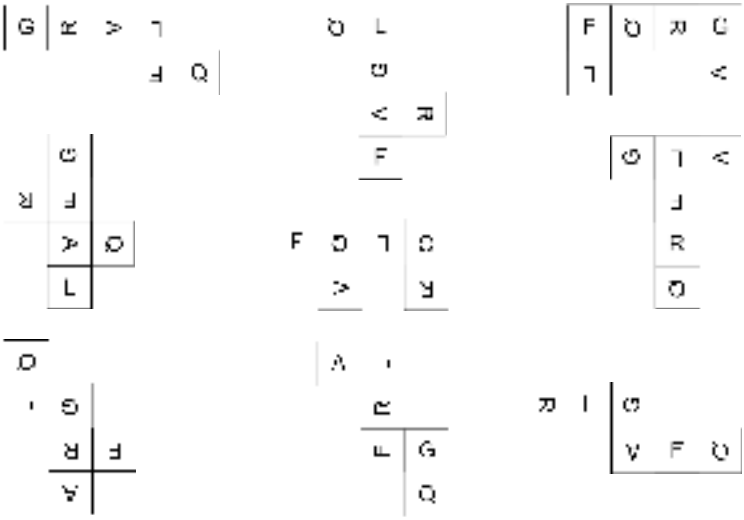
Benefits of the game:

Active work of the class. Development of pupils' spatial imagination. Becoming familiar with a cube net in various forms.

Demonstration 11 Assignment for the *Cube with Letters* game

The assigned cube

Find nets of the assigned cube in these pictures



11 Magic Square

Thematic area:

This game is suitable for mathematical areas dealing with the *Addition and Subtraction of Numbers and Expressions*, and for the subject matter dealing with the *Divisibility of Non-negative Numbers*.

Educational targets:

To practice the addition and subtraction of numbers and expressions. To practice the divisibility of non-negative numbers. Feedback on pupils' mastering of the mentioned knowledge. The game also develops combinatorial thinking of pupils and their cooperation.

Game environment:

Pupils and the teacher: The class, pairs sitting at desks cooperate. The teacher has an organizational and controlling role.

Material environment: Incomplete assignments of a magic square (or various magic squares) for each pair. The magic square is a table of $n \times n$ boxes containing numbers the sum of which in all rows, columns and both diagonals is equal – see *Demonstration 12.*

Game duration: 5–15 min

Demonstration 12 Magic square containing integers.

1	0	-7
-10	-2	6
3	-4	-5

Game procedure:

At the start we distribute an incomplete assignment of the magic square (*Demonstration 13*) to pupils at each desk. The pupils' task is to fill in the missing data so that the resulting square is magic (equal sums of lines, columns and diagonals). After the end of the work the teacher will collect the assignments, which he/she will check on the basis of the prepared correct results. The aim of the pairs sitting at desks is to fill in as much correct data as possible.

In the variant of the game dealing with the divisibility of integral numbers, the magic square represents a table of numbers the sum of which in all rows, columns and both diagonals is divisible by the given natural number. In this case there are several correct solutions when filling in the data.

Note:

Construction of the assignment of a magic square sized 3 x 3 for adding and subtracting numbers and expressions: When constructing such magic square, we will start with the central box. To make the magic square solvable, we need to write

3 more values in some of the lines or columns. Another possibility is that in addition to the central box, we will also write 2 more values in one diagonal and one more value in any line. In both cases we must meet the condition that the sum of the data centrally symmetrical with regard to the centre of the magic square is double the value in the central box; the sum of the whole line or column will be triple the value of the central box.

Demonstration 13 Assignments of magic squares

1) Assignment of a magic square for the arithmetic of integral numbers

-1	0	7
	2	

2) Assignment of a magic square for adding and subtracting expressions

$-2y$		
x^2	$x^2 - y$	
		$2x^2$

3) Assignment of a magic square for the divisibility by 3

10		7
	6	

Final assessment:

Both pupils at the desk will get a point for the activity for each correct entry.

Benefits of the game:

Active work of the class, inner motivation of pupils thanks to the attractiveness of the task assignments, as well as the competitiveness among desks. Development of the ability to cooperate.

12 Jigsaw

Thematic area:

This game is suitable for the thematic area of *Angle and Its Dimension* and for the thematic areas of *Triangle* and *Parallelogram and Trapezoid.*

Educational targets:

To develop the pupils' ability to distinguish important characteristics of plane geometric shapes. Feedback for the pupils on mastering the subject matter.

Game environment:

Pupils and the teacher: Pairs play. The teacher has an organizational and controlling role.

Material environment: Pack of cards and the game board for each team (see *Demonstration 14*).

Game duration: 5–10 min

Game procedure:

At the start we distribute a pack of cards and a game board to each team. The mentioned pack consists of cards where the geometric shapes of certain characteristic features are depicted. These features are shown in the game board boxes. The task of the players is to correctly match each card to a box with the characteristic features of the object depicted on the given card.

Final assessment:

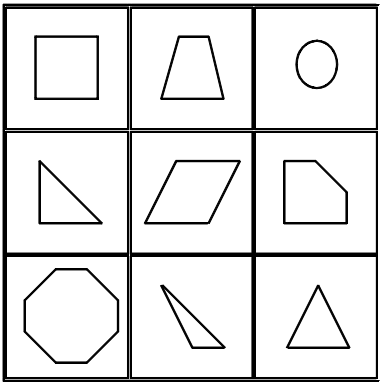
The teacher will check the correctness of the solution. Pupils will get a certain number of points for the activity for each correctly matched card.

Benefits of the game:

Active work of the class. Development of the pupils' ability to perceive the important characteristics of plane geometric shapes. Cooperation of pairs at desks.

Demonstration 14 Game board and cards for the *Jigsaw* game in the thematic area of *Angle and its Dimension*

4 right angles	2 obtuse angles 2 acute angles	no angle
1 right angle 2 acute angles	2 acute angles 2 obtuse angles	3 right angles 2 obtuse angles
6 obtuse angles	1 obtuse angle 2 acute angles	3 acute angles



13 Pyramid Construction

Thematic area:

The game is suitable for thematic areas dealing with the *Arithmetic of Numbers and Expressions*, in particular their adding and subtracting.

Educational targets:

To practice the addition and subtraction of numbers and expressions. To develop the pupils' ability to count carefully in order to get correct results. Feedback on the quality of knowledge, both for the teacher and the students. Development of the ability to cooperate.

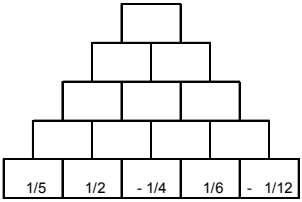
Game environment:

Pupils and the teacher: Pairs at desks cooperate. The teacher has an organizational and controlling role.

Material environment: Each pair of pupils will get a partially filled-in pyramid scheme (see *Demonstration 15*).

Game duration: 5–10 min

Demonstration 15 Pyramid scheme to practice adding fractions



Game procedure:

Pairs at desks must correctly fill in the pyramid scheme. When filling it in, there is a rule that the entry in a higher pyramid line is the sum of the two entries in adjacent boxes of the line below.

If we write all entries in the bottom line, the pupils will only use adding. If we write an incomplete bottom line and we also write some entries in higher lines, the pupils will also have to use subtraction when filling in the pyramid (*Demonstration 16*).

The aim of the pupils is not to make any mistakes when calculating, and so to get a correct value in the top pyramid line. On the basis of this entry, the teacher will check if the pyramid has been filled in correctly.

Final game assessment:

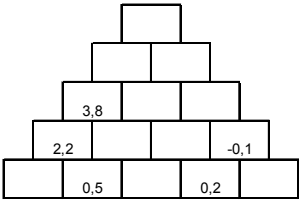
Both pupils in a pair will get a certain number of points for the activity for a correctly filled-in pyramid (depending on the difficulty of tasks).

Benefits of the game:

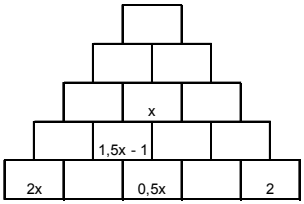
Active work of the class. Attractiveness of the game assignment motivates pupils. This game makes pupils pay more attention when counting, as the overall solution success depends on each result. It is advantageous for the teacher, as checking the pupils' work results is simple.

Demonstration 16 Pyramid schemes to practice adding and subtracting

1) Scheme for practicing adding and subtracting decimal numbers



2) Pyramid for practicing adding and subtracting expressions



INDIVIDUAL GAMES

Games for Individuals

14 The Way Home

Thematic area:

This game is suitable for the thematic area of the *Divisibility of Positive Integers*.

Educational targets:

To practice divisibility criteria, feedback on mastering the subject matter.

Game environment:

Pupils and the teacher: Pupils work independently. The teacher has an organizational and controlling role.

Material environment: Table of figures for each pupil (*Demonstration 17*).

Game duration: 15–30 min

Game procedure:

Pupils are given a table of figures with the entrance (the start of the journey) and exit (home) marked. The task of the pupils is to find a way to connect the start of the journey with the home. The pupils can only move over numbers divisible by the given number (the table in *Demonstration 17* serves for the criteria of divisibility by 2, 3 and 6), only in the horizontal and vertical direction, never diagonal. The pupils write the course of the journey, i.e. the sequence of numbers which

they have passed, on a special sheet. The aim is to find the correct way home, or if they exist, as many correct ways home as possible. The teacher will assess the correctness of the solution on the basis of the collected journey entries.

Final assessment:

Pupils will get a certain number of points for the activity for each correct entry of the way home.

Benefits of the game:

Inner motivation of pupils by the attractiveness of the game's assignment. Active work of the class. Feedback on the pupils' knowledge level.

Demonstration 17 Table of figures for *The way home* game

174	9	51	135	18	26	39	54	44	18
36	25	39	18	21	156	81	27	333	31
84	12	42	82	36	57	63	54	32	35
8	127	78	99	204	111	9	303	49	108
144	16	18	102	96	6	47	36	105	42
72	64	6	101	44	60	103	261	77	51
11	98	19	24	67	24	25	222	29	36
45	106	15	21	108	48	132	30	72	168
33	24	12	66	94	13	27	42	90	24
18	72	55	14	22	18	38	204	108	54

15 Spot the Mistake!

Thematic area:

This game is suitable for all mathematical areas.

Educational targets:

To practice and strengthen pupils' knowledge in different areas of mathematics. To develop the ability to assess the correctness of solutions of mathematical tasks. Feedback on the quality of knowledge both for the teacher and the students.

Game environment:

Pupils and the teacher: Pupils work independently. The teacher has an organizational and controlling role.

Material environment: Each pupil gets an assignment containing solved mathematical tasks (*Demonstration 18*).

Game duration: 5–40 min

Game procedure:

Each pupil gets an assignment containing solved mathematical tasks. There are mistakes in the solution procedure of these tasks. The aim is to spot and correct as many mistakes as possible.

Final game assessment:

After the end of the game the teacher will collect the assignments and assess the pupils' work. Pupils will get a certain number of points for the activity for each correctly corrected mistake (the number of points depends on the difficulty).

Benefits of the game:

Active work of all pupils in the class. Motivation of pupils by the effort to achieve as many points as possible. The game develops the ability of pupils to assess the solutions of math tasks; through this game the pupils realize the most frequent mistakes which are made, and that helps them eliminate them.

Demonstration 18 Assignment of the task for the *Spot the Mistake!* game related to the thematic unit of *Area of geometric shapes*

Task:
We paint rectangular wall with dimensions 4 m and 2,5 m. How many cans of paint we need to buy? We know, that one can of paint is enough to paint 5 m^2 of the wall. (Solve this word problem.)

Solution:

We know		**Calculation**	
dimensions of the wall:	*4 m and 2,5 m*	$A = 4 \cdot 2,5$	2,5
1 can of paint is enough to paint	*5* m^2	$A = 100$	x 4
			100
		$100 \cdot 5 = 500$	

Answer

We need 500 cans of paint.

16 Symmetric Pictures

Thematic area:

This game is suitable for the thematic area of *Central and Axial Symmetry*.

Educational targets:

To practice displaying through axial symmetry. Feedback on mastering the subject matter. Development of pupils' imagination.

Game environment:

Pupils and the teacher: Pupils work independently. The teacher has an organizational and controlling role.

Material environment: Incomplete pictures in a square grid for each pupil. (*Picture 5.4*)

Game duration: 15–20 min

Game procedure:

The task of the pupils is to finish the pictures placed on the graph paper. The missing part of the picture is axially symmetric with the assigned part in line with the marked axis of symmetry.

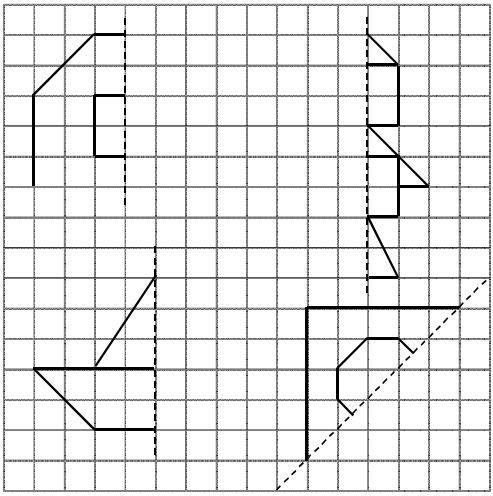
Final assessment:

The pupils will get a certain number of points for the activity for each correctly finished picture (depending on the difficulty of the picture).

Benefits of the game:

Active work of all pupils. Practicing central symmetry in an interpretation attractive for pupils. Feedback on the level of

knowledge both for the teacher and the pupils. Development of pupils' imagination.



Pic. 5.4 Demonstration of the assignment for the *Symmetric Pictures* game

17 Hidden Exercises

Thematic area:

This game is suitable for thematic areas dealing with the *Arithmetic of Numbers and Expressions* and the thematic area of *Powers and Roots*.

Educational targets:

To practice arithmetic operations with numbers and simple expressions. Feedback on the quality of knowledge both for the teacher and the students.

Game environment:

Pupils and the teacher: Pupils work independently. The teacher has an organizational and controlling role.

Material environment: Numeric table for each pupil (*Picture 5.5*). The table can also be prepared on the blackboard or projected with an overhead projector. Sheet of paper for each pupil.

Game duration: 5–10 min

2	3	8	3	5	-1
11	2	121	3	7	2
1/2	4	125	27	2	1
-5	2	-2	1	49	77
64	16	1/4	3	3	-1/2
1/2	3	1/8	36	-8	1

Pic. 5.5 Numeric table for the *Hidden Exercises* game for the subject matter of *powers and roots*, in particular focused on squares and cubes

Game procedure:

The task of the pupils is to find trios of numbers or expressions in lines, columns and diagonals. There is a rule for each trio: the result of a numeric operation whose arguments are the first two data will be the third datum in the sequence. Operation can be adding, subtracting, multiplying, dividing or raising the first entry to the power of the second entry, etc. Pupils will write the identified trios in the form of a mathematic operation on the sheet of paper. The task is to find as many 'hidden exercises' as possible. Example of the game – see *Demonstration 19*.

Final game assessment:

Pupils will obtain one point for the activity for each identified exercise.

Benefits of the game:

Practicing routine numeric operations in an interesting way for pupils. Active work of the class. Motivation of pupils by the attractiveness of the game's context as well by competitiveness.

Demonstration 19 Pupils' solution of the *Hidden Exercises* game for the thematic unit of *Powers and roots*

2	3	8	3	5	-1
11	2	121	3	7	2
1/2	4	125	27	2	1
-5	2	-2	1	49	77
64	16	1/4	3	3	-1/2
1/2	3	1/8	36	8	1

Identified exercises:

$$2^3 = 8,\ 11^2 = 121,\ \left(\frac{1}{2}\right)^3 = \frac{1}{8},\ 4^2 = 16,\ 3^3 = 27,\ 7^2 = 49,\ (-1)^2 = 1,$$

$$\left(\frac{1}{2}\right)^2 = \frac{1}{4},\ (-2)^3 = -8,\ 1^3 = 1,\ 5^3 = 125.$$

Pair games

18 3D Noughts and Crosses

Thematic area:

The game is focused on the *Development of Spatial Imagination of Pupils.*

Educational targets:

Development of pupils' spatial imagination. The game develops pupils' combinatorial and strategic thinking.

Game environment:

Pupils and the teacher: Pairs at desks play. The teacher has an organizational and controlling role.

Material environment: Set of 4 square grids sized 4 x 4. The set is formed by single cube layers, marked as 'floors', drawn in the game board *(Picture 5.6*).

Game duration: 5–15 min

Game procedure:

At the start, we distribute a sheet of paper to pairs containing sets of square grids representing cube 'floors'. The pupils will take turns to draw their symbols (0 or X) on a blank box of any square grid of the given set. The aim of the players is to achieve that 4 of his/her symbols are placed after each other horizontally, vertically or diagonally either on one floor or across all 4 floors. The first of the players to achieve it is the winner. Example of the game – see

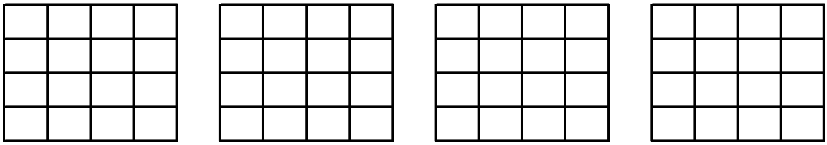
Demonstration 20. In the following game, the pupils will change the order in which they started the previous game.

Final assessment:

Players play more games. The lowest number is 2 to make sure that each player has started the same number of games. Each pair will put down the mutual score and submit the record to the teacher. Both the winner and the loser will get a certain number of points for the activity for each game (e.g. 3 points for the winner, 1 point for the loser).

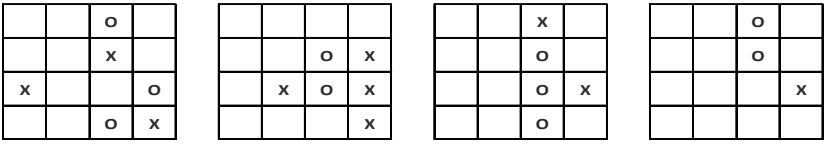
Benefits of the game:

Development of spatial imagination in a playful form. Active work of the whole class, inner motivation of pupils through competitiveness.



Pic. 5.6 Set of square grids for the '3D Noughts and Crosses' game

Demonstration 20 A sample *3D Noughts and Crosses* game



The player with symbol O wins

19 Bard

Thematic area:

This game is suitable for the thematic area of *Divisibility of Positive Integers*.

Educational targets:

To practice divisibility by 3. Feedback for the pupils on mastering the subject matter. The game develops pupils' combinatorial and strategic thinking.

Game environment:

Pupils and the teacher: Pairs at desks play. The teacher has an organizational and controlling role.

Material environment: Square grid sized 3 x 3 for each pair (*Picture 5.7*).

Game duration: 5–10 min

Game procedure:

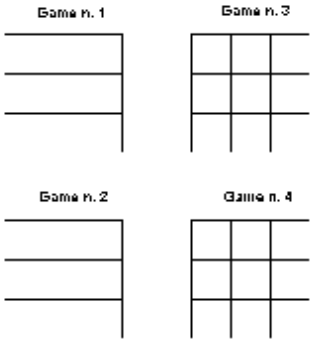
At the start pairs are given a sheet of paper with square grids sized 3 x 3. The players take turns and write any integer numbers from 1 to 9 into blank boxes of the grid. Each of the numbers may only be written once. Having filled in the grid, the pupils will determine which of the three-digit numbers formed by the rows and columns of the table are divisible by 3. The player who has started the game will get a point for each such 3-digit number divisible by 3, the second player for each number non-divisible by 3. The player with the highest number of points is the winner. In the next game the order of the players will change. Result of one game – see *Demonstration 21*.

Final assessment:

Players play more games. The lowest number is 2 to make sure that each player has started the same number of games. Each pair will put down the mutual score and submit the record to the teacher. Both the winner and the loser will get a certain number of points for the activity for each game (e.g. 3 points for the winner, 1 point for the loser).

Benefits of the game:

Active work of the class, inner motivation of pupils through competitiveness. The pupils' mutual cross-checking eases the load on the teacher.



Pic. 5.7 Game boards for the *Bard* game

Demonstration 21 A sample result of the *Bard* game

3	1	8	divisible
5	9	6	indivisible
4	2	7	indivisible
divisible	divisible	divisible	

The first player wins 4 : 2

20 Dim

Thematic area:

This game is suitable for the thematic area of *Divisibility of Positive Integers*.

Educational targets:

To practice determining the divisibility of positive integers. Feedback for the pupils on mastering the subject matter. The game develops pupils' combinatorial and strategic thinking.

Game environment:

Pupils and the teacher: Pairs at desks play. The teacher has an organizational and controlling role.

Material environment: A certain number of beans or other suitable small objects for each pair.

Game duration: 10–20 min

Game procedure:

At the start we give a certain number of beans to pairs (e.g. 35 beans, pupils can bring them from home). The pupils will take turns and take a certain number of beans from the

heap, while they can only take the number corresponding to a non-negative integer higher than 1 which is the divisor of the current number of beans in the heap. One sample game – see *Demonstration 22*. The player who has to take the last bean is the loser. In the following game, the pupils will change the order in which they started the previous game.

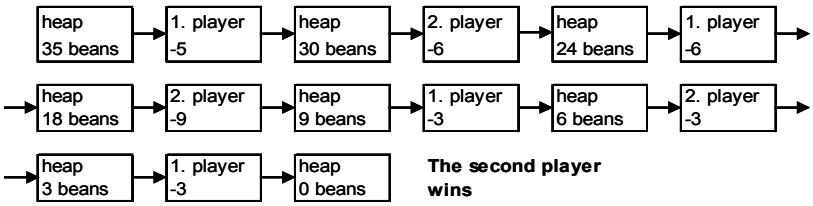
Final assessment:

Players play more games. The lowest number is 2 to make sure that each player has started the same number of games. Each pair will put down the mutual score and submit the record to the teacher. Both the winner and the loser will get a certain number of points for the activity for each game (e.g. 3 points for the winner, 1 point for the loser).

Benefits of the game:

Active work of the class, inner motivation of pupils through competitiveness. The pupils' mutual cross-checking eases the load on the teacher.

Demonstration 22 A sample *Dim* game



21 Domino

Thematic area:

The domino game is suitable for thematic areas dealing with the *Conversion of Units of Area* and the *Conversion of Units of Volume,* and also for thematic units dealing with the *Arithmetic of Decimal Numbers* and the *Arithmetic of Rational Numbers.*

Educational targets:

To practice the conversion of area or volume units or to practice arithmetic operations with decimal numbers or rational numbers. Feedback for the pupils on mastering the subject matter. The game develops the pupils' ability to mutually check their activity.

Game environment:

Pupils and the teacher: The class, pairs sitting at desks play. The teacher has an organizational and controlling role.

Material environment: Set of domino tiles for each pair of pupils (see *Demonstration 23*).

Game duration: 15–20 min

Game procedure:

At the start we distribute a set of domino bones to pairs of pupils at desks. For thematic areas dealing with the conversion of area or volume units, the bones will contain pairs of data of these quantities, stating the numeric value and the unit. The domino bones are shuffled and placed with the numbers facing down. One of the bones is turned over. Then the pupils take turns, i.e. they turn over one of the

previously unturned bones. If it can be added to other placed bones, the pupil can do it (but not necessarily), otherwise he/she has to keep it. Only bones with the same area/volume value on the added side can be added (these values are stated in different units). The winner is the player with the lowest number of bones which cannot be added to other bones on the table at the end of the game.

If you want to use the game for areas dealing with the arithmetic of decimal numbers or the arithmetic of rational numbers, use domino bones containing the entries of adding, subtracting, multiplying or dividing 2 numeric values. The bones are added to each other so that the result of the operation on one bone is the first one of the numeric data on the other bone (*Demonstration 24*).

Final assessment:

Each pair will put down the mutual score and submit the record to the teacher. Both the winner and loser will obtain a certain number of points for the activity for each game (e.g. 3 points for the winner, 1 point for the loser. In the case of a draw, both players will get 1 point).

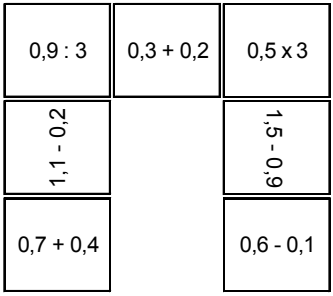
Benefits of the game:

Active work of the class, inner motivation of pupils through competitiveness. The pupils' mutual cross-checking eases the load on the teacher.

Demonstration 23 Domino bones used in the thematic unit of *Area of Geometric Shapes*

1 m^2	500 cm^2	5 dm^2	2 cm^2	200 mm^2	10 dm^2	0,1 m^2	3 cm^2	1 000 000 m^2	0,000 3 m^2
0,04 dm^2	100 dm^2	4 cm^2	2 dm^2	0,02 m^2	1 ha	100 a	0,000 002 m^2	2 mm^2	1 km^2
1 m^2	4 cm^2	0,04 dm^2	1 000 000 m^2	1 km^2	3 cm^2	0,000 3 m^2	200 mm^2	100 a	2 cm^2
500 cm^2	100 dm^2	5 dm^2	0,02 m^2	2 dm^2	0,1 m^2	10 dm^2	0,000 002 m^2	2 mm^2	1 ha
0,000 002 m^2	100 dm^2	1 m^2	0,04 dm^2	4 cm^2	1 ha	100 a	2 cm^2	2 mm^2	3 cm^2
0,000 3 m^2	10 dm^2	0,1 m^2	2 dm^2	0,02 m^2	1 000 000 m^2	1 km^2	500 cm^2	200 mm^2	5 dm^2
1 000 000 m^2	100 dm^2	1 m^2	2 mm^2	0,000 002 m^2	500 cm^2	5 dm^2	100 a	1 ha	2 cm^2
1 km^2	4 cm^2	0,04 dm^2	0,000 3 m^2	3 cm^2	0,01 m^2	10 dm^2	2 dm^2	0,02 m^2	200 mm^2

Demonstration 24 Domino bones serving to practice the *Arithmetic of Decimal Numbers*



22 Enades

Thematic area:

This game is suitable for the thematic area of *Powers and Roots*.

Educational targets:

To practice determining the powers of non-negative numbers by heart. Feedback for the pupils on mastering the subject matter. The game develops pupils' combinatorial and strategic thinking.

Game environment:

Pupils and the teacher: Pairs at desks play. The teacher has an organizational and controlling role.

Material environment: Sheet of paper for each pair.

Game duration: 5–10 min

Game procedure:

The players deduct any powers of 2, 3 or 5 with an exponent of a positive integer from an initial number n (e.g. $n = 100$). Both players take turns and write down the status of the game on the sheet of paper. The player who gets *0* or *1* in his/her turn wins. In the following game, the pupils will change the order in which they started the previous game. A sample game – see *Demonstration 25*.

Final assessment:

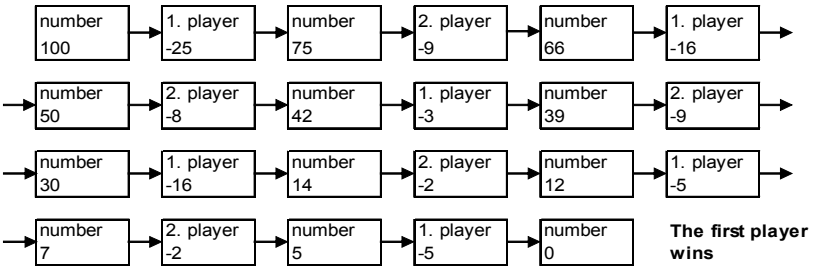
Players play more games. The lowest number is 2 to make sure that each player has started the same number of games. Pupils will put down the mutual score and submit the record to the teacher. Both the winner and the loser will get

a certain number of points for the activity for each game (e.g. 3 points for the winner, 1 point for the loser).

Benefits of the game:

Active work of the class, inner motivation of pupils through competitiveness. The pupils' mutual cross-checking eases the load on the teacher.

Demonstration 25 A sample *Enades* game



23 Snake

Thematic area:

This game is suitable for the thematic area of *Central and Axial Symmetry*.

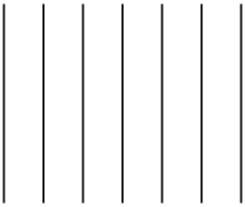
Educational targets:

To practice displaying a point through central symmetry. Pupils' motivation. Feedback on mastering the subject matter. To develop pupils' imagination and strategic thinking.

Game environment:

Pupils and the teacher: Pairs of pupils at desks play. The teacher has an organizational and controlling role.

Material environment: Square grid for each pair (*Picture 5.8*).



Pic. 5.8 Square grid for the *Snake* game

Game duration: 15–20 min

Game procedure:

The teacher distributes game boards to the pupils. The pupil who is starting will draw a circle in any square of the game grid and write the number 1 in it. (The snake has been born and is 1 day old.) The second player will choose any grid point, i.e. a point where the lines of the square grid intersect, and through the central symmetry with the centre in this point he/she will display the snake in its new position. He/she will draw another circle in any free square adjacent to the new snake's position. He/she will write the number 2 in both of these circles. (The snake has moved and grown and is 2 days old.) They continue analogically, each next turn represents moving the snake and making it longer by one square of the grid. The players take turns. If any part of the snake is to be displayed beyond the square grid or in the space where the snake has been, the player loses. Any player also loses if he/she has no free space to make the snake longer. In the following game, the pupils will change the order

in which they started the previous game. A sample game – see *Demonstration 26*.

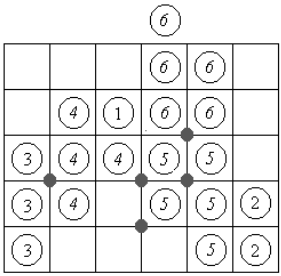
Final assessment:

Players play more games. The lowest number is 2 to make sure that each player has started the same number of games. Pupils write the scores of mutual games. They will get a previously agreed number of points for the activity both for winning or losing (e.g. 3 points for winning, 1 point for losing).

Benefits of the game:

Practicing the subject matter in a context attractive for pupils. Inner motivation of pupils through competitiveness. Active work of the whole class. Feedback on the knowledge level for the pupils. Development of strategic thinking and imagination (when planning draws).

Demonstration 26 A sample *Snake* game



The first player wins

24 Treasure Seekers

Thematic area:

This game is suitable for the thematic area of *Functions*.

Educational targets:

Propaedeutic of point coordinates in a rectangular coordinate system in a plane.

Game environment:

Pupils and the teacher: Pairs at desks play. The teacher has an organizational and controlling role.

Material environment: Square grid for each pupil (*Picture 5.9*).

Game duration: 15–20 min

Game procedure:

At the start of the game, each pupil draws "treasures" in his/her square grid. Each player has to draw a determined number of treasures and the drawn treasures must not touch, i.e. one treasure can not be drawn in any grid squares adjacent to a different treasure (see *Demonstration 27*). Pupils take turns during the game. In each turn they will say an ordered pair of non-negative integers identifying the coordinates of the square where the player is "looking for the treasure". The first coordinate represents the number of the rows, the second the number of the column in the square grid. The opponent will say if he/she has a treasure drawn in the stated square. If the player has identified all the squares representing the given treasure, the opponent will announce

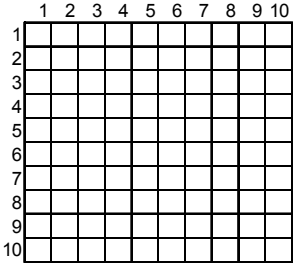
its finding. The player who has found all the treasures in the opponent's game board is the winner.

Final assessment:

Each pair will put down the mutual score and submit the record to the teacher. Both the winner and the loser will get a certain number of points for the activity for each game (e.g. 2 points for the winner, 1 point for the loser).

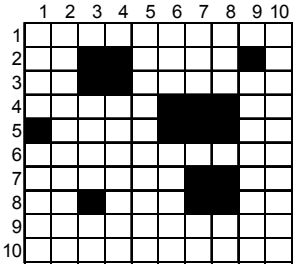
Benefits of the game:

Coordinate propaedeutic in an attractive form for pupils. Active work of the class. The attractiveness of the game's context and competitiveness motivate the pupils.



Pic. 5.9 Square grid for the *Treasure Seekers* game

Demonstration 27 Layout of 'treasures' in the *Treasure Seekers* game



25 Logic

Thematic area:

The game can be used for the thematic area of *Combinatorics.*

Educational targets:

To develop pupils' logical and combinatory thinking.

Game environment:

Pupils and the teacher: Pairs at desks play. The teacher has an organizational and controlling role.

Material environment: The game only requires paper and writing utensils, no other special aids.

Game duration: 10–20 min

Game procedure:

One of a pair of students writes 5 numbers next to each other, selected from a set of non-negative integers from 1 to 8, while each of these 8 numbers can only be written. The other pupil should guess this quintuple. He/she will write his/her supposed quintuple on his/her sheet of paper. The pupil who has selected the numbers will say the number of the correctly guessed numbers, and the number of numbers whose location has also been guessed correctly. The other pupil continues guessing, until he/she finds the whole quintuple, while he/she has 10 attempts to make a guess. The pupils will swap their roles in the next game. A sample of the game is shown in *Demonstration 28*.

The difficulty of the game can be changed by changing the number of sought numbers and the set of non-negative integers from which we are choosing the stated numbers;

e.g. by guessing 4 numbers selected from the set of non-negative integers from 1 to 6.

Final game assessment:

Pupils put down the game procedures. Each guessing pupil will get a certain number of points for the activity for every correctly guessed quintuple of numbers.

Benefits of the game:

Active work of the class. Development of pupils' logical thinking in an activity attractive for pupils.

Demonstration 28 A sample *Logic* game

	guessed quintuple	*n. of correctly guessed numbers*	*n. of numbers whose location has also been guessed correctly*
1.	1 3 5 8 7	3	0
2.	7 5 1 2 4	4	2
3.	7 5 1 6 4	3	2
4.	7 5 2 6 1	3	1
5.	7 2 5 8 4	3	3
6.	**7 2 3 1 4**	**5**	**5**

26 Mathematical Memory

Thematic area:

This game is suitable for thematic areas dealing with the *Arithmetic of Numbers and Expressions.*

Educational targets:

To train counting 'from memory' in different numeric domains or with expressions. Feedback for the pupils on mastering the subject matter. The game develops pupils' spatial memory.

Game environment:

Pupils and the teacher: Pairs at desks play. The teacher has an organizational and controlling role.

Material environment: Set of memory cards for each pair (see *Demonstration 29*).

Game duration: 15–20 min

Game procedure:

At the start we distribute a pack of memory cards to pairs. This pack consists of cards containing the assignments of simple tasks and cards with the results of these tasks. The stated tasks have to be solvable 'from memory'. The cards are shuffled and placed with the data facing down. One of the pupils turns over any two cards. If it is a matching pair – the task assignment and its solution – the pupil will keep the cards and continue turning over. If these two cards do not match, they are turned back and the other player continues. The winner is the player who has the higher number of obtained matches after all the cards have been turned over.

The game can be simplified by playing separately with 2 packs of cards. One pack contains the assignments of tasks, the other pack contains the results of these tasks. (The packs can be distinguished, e.g. by the colour on the back of the cards.) When it is one pupil's turn, he/she will turn over one card in each pack. This game variant is less demanding for the memory.

Final assessment:

Each pair will put down the mutual score and submit the record to the teacher. Both the winner and the loser will obtain a certain number of points for the activity for each game (e.g. 3 points for the winner, 1 point for the loser. In the case of a draw, both players will get 1 point).

Benefits of the game:

Active work of the class, inner motivation of pupils through competitiveness. The pupils' mutual cross-checking eases the load on the teacher.

Demonstration 29 Memory cards used in the thematic area of *Decimal Numbers*

1) Task assignments

0,1 + 0,9	1,5 - 0,3	0,5 x 2	1,2 : 2
2,2 - 0,4	1,2 + 0,5	0,7 : 7	3 x 0,1
0,2 x 2	3 - 2,8	2 : 4	1,2 + 0,8
1 : 2	0,1 + 0,7	1 - 0,6	2 x 0,9

2) Solutions of the tasks

1	1,2	1	0,6
1,8	1,7	0,1	0,3
0,4	0,2	0,5	2
0,5	0,8	0,4	1,8

27 Powers

Thematic area:

This game is suitable for the thematic area of *Powers and Roots*.

Educational targets:

It is suitable to use this didactic game when teaching the thematic area dealing with powers and roots. The aim of the game is that pupils remember the squares of selected non-negative integers and practice counting with them. Another aim of the game is to develop pupils' strategic thinking and data analysis.

Game environment:

Pupils and the teacher: Pairs at desks play. The teacher has an organizational and controlling role.

Material environment: Sheet of paper for each pair.

Game duration: 2–5 min

Game procedure:

Pupils are divided into pairs. They write 2 columns of non-negative integers from 1 to 9 with the square symbol on a sheet of paper. First player A deducts a square of one of the numbers stated in the columns from the number 100. Then he/she will cross out the used number. Then player B deducts a square of any uncrossed number stated in the columns from the number which was a result of player A, and he/she will also cross out the used number. The players take turns until a player is forced to deduct a square of such number that the result will be negative. This player will lose. In the following game, the pupils will change the order in which they started the previous game. A sample game – see *Demonstration 30*.

Demonstration 30 A sample *Powers* game

~~1^2~~	~~1^2~~	100	
2^2	2^2	-81	Player A
3^2	3^2	19	
4^2	~~4^2~~	-16	Player B
5^2	5^2	3	
6^2	6^2	-1	Player A
7^2	7^2	2	
8^2	8^2	-1	Player B
~~9^2~~	9^2	1	

The player A is forced to deduct a square of such number that the result will be negative. So, the player B wins.

Final assessment of pupils' work:

Players play more games. The lowest number is 2 to make sure that each player has started the same number of games.

Each pair will put down the mutual score and submit the record to the teacher. Both the winner and the loser will get a certain number of points for activity for each game (e.g. 2 points for the winner, 1 point for the loser).

Benefits of the game:

Work with the subject matter in a context attractive for pupils. Motivation through competitiveness. Development of strategic thinking.

28 Letter 'L' Travelling

Thematic area:

This game is suitable for the thematic area of *Central and Axial Symmetry*.

Educational targets:

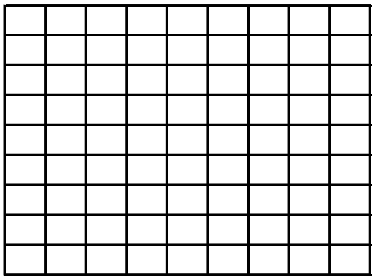
To practice displaying a shape through central symmetry. Feedback on mastering the subject matter. To develop pupils' imagination and strategic thinking.

Game environment:

Pupils and the teacher: Pairs at desks play. The teacher has an organizational and controlling role.

Material environment: Game board for each pair (*Picture 5.10*).

Game duration: 15–20 min



Pic. 5.10 Game board for the *Letter 'L' Travelling* game

Game procedure:

The teacher distributes game boards to the pupils. The first pupil will draw a letter L in the game board, formed by 4 squares of the game board. The second player will choose any grid point, i.e. a point where the lines of the square grid intersect, and through the central symmetry with the centre in this point he/she will display the L in its new position. The player who cannot place the letter L in the blank squares of the game board will lose. A sample game – see *Demonstration 31*. In the following game, the pupils will change the order in which they started the previous game.

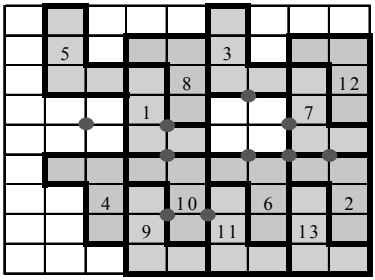
Final assessment:

Players play more games. The lowest number is 2 to make sure that each player has started the same number of games. Pupils write the scores of mutual games. They will get a previously agreed number of points for the activity both for winning or losing (e.g. 3 points for winning, 1 point for losing).

Benefits of the game:

Practicing a subject matter in a context attractive for pupils. Inner motivation of pupils through competitiveness. Active work of the class. Feedback for the pupils on mastering the central symmetry. Development of strategic thinking and imagination (when planning draws).

Demonstration 31 A sample *Letter L Travelling* game



The first player wins

29 Number of Divisors

Thematic area:

This game is suitable for the thematic area of *Divisibility of Non-negative Integers*.

Educational targets:

To practice determining the divisibility of non-negative integers. Feedback for the pupils on mastering the subject matter. The game develops pupils' combinatorial and strategic thinking.

Game environment:

Pupils and the teacher: Pairs at desks play. The teacher has an organizational and controlling role.

Material environment: A blank sheet of paper for each pair.

Game duration: 5–10 min

Game procedure:

During the game the players deduct any non-negative number from 1 to 5 from an initial non-negative number n (e.g. $n = 30$). After the deduction, the player will find out how many divisors the originated number has; the number of divisors determines the number of points which the player will get for his/her draw. The player will write the originated number as well as the number of divisors on a sheet of paper. Players take turns until they get zero. The player who has achieved the most points in the game is the winner. In the next game the pupils will swap the order in which they started. A sample game – see *Demonstration 32*.

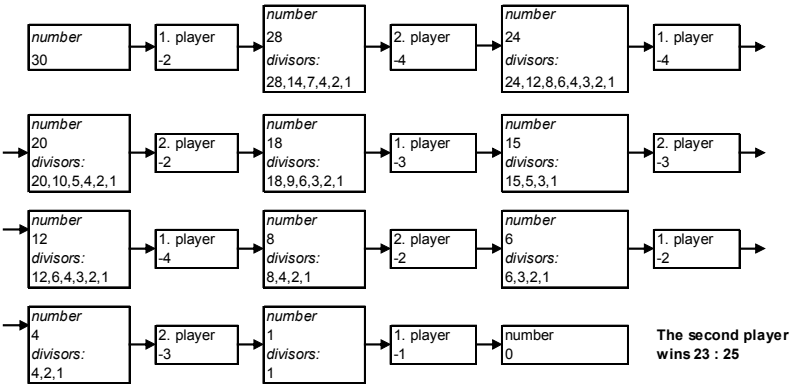
Final assessment:

Players play more games. The lowest number is 2 to make sure that each player has started the same number of games. Each pair will put down the mutual score and submit the record to the teacher. Both the winner and the loser will get a certain number of points for the activity for each game (e.g. 3 points for the winner, 1 point for the loser).

Benefits of the game:

Active work of the class, inner motivation of pupils through competitiveness. The pupils' mutual cross-checking eases the load on the teacher.

Demonstration 32 A sample *Number of Divisors* game



30 Equations

Thematic area:

This game is suitable for the thematic area of *Solving Linear Equations and their Systems*.

Educational targets:

This game is suitable for training linear equations and the general improvement of pupils' understanding of equations and their meaning. The analysis of the game's strategy is quite simple and thus also suitable for younger pupils. In a simple example, it enables us to demonstrate the concept of a winning strategy and the power of logical thinking.

Game environment:

This game is not suitable to organize tournaments. Pairs at desks play, while the number of games depends on the period in which pupils are able to find a winning strategy. The teacher has an organizational and controlling role and helps pupils with the strategy analysis.

Material environment: The game is played in a pattern displayed in *Picture 5.11*. The number of equations may naturally be modified if necessary.

Game duration: 5–10 min

...=...
...+...=...
...+...+...=...
...+...+...+...=...
...+...+...+...+...=...
...+...+...+...+...+...=...

Pic. 5.11 Game board of the *Equations* game

Game procedure:

The game is played by 2 pupils. When it is their turn, the players write any integral number in the blank spaces marked by 3 points in the game board. They swap, while the first player wins if all the equations are correct at the end of the game. The second player wins if at least one of the equations is not correct. In the following game, the pupils will change the order in which they started the previous game. Example of the game – see *Demonstration 33*.

Final assessment of pupils' work:

This game is illustration of strategy analysis. Therefore, we do not award points.

Strategy analysis:

When playing, the pupils must realize that they can fill in integral numbers in any blank spaces. Then the winning strategy of the first player in the game board (shown in Picture 5.11) is the filling in of the last blank space in the equation so that the equation is correct. The pupils can be asked how the number of equations will change the winning strategy of the game. This game can be modified so that pupils also fill in operators, and depending on difficulty we can also change the set of numbers which the numbers are filled in from.

Benefits of the game:

Practicing the meaning of equations in a playful context. The opportunity independently find a winning strategy for a mathematical game. Clarifying the winning strategy notion. Development of strategic and logical thinking.

Demonstration 33 A sample *Equations* game

$$1 = 1$$
$$(-7) + 2 = -5$$
$$4 + 3 + 4 = 11$$
$$1 + 3 + 5 + 2 = 11$$
$$5 + 2 + 3 + (-1) + 9 = 18$$
$$1 + 4 + 8 + 2 + 7 + (-7) = 15$$

6 TABLE OF DIDACTIC GAMES

THEMATIC AREA	NAME AND TYPE OF DIDACTIC GAME: CM = cooperative game for multiple teams C2 = cooperative game for teams of 2 GI = games for individuals PG = pair games		
Universal games (suitable for most thematic areas)	Bingo	CM	page 55
	Circles	CM	page 58
	Puzzle	CM	page 60
	Lines Contest	CM	page 65
	Chairs Contest	CM	page 67
	Cipher game	CM	page 70
	Relay	CM	page 77
	Spot the Mistake!	GI	page 91
Algebraic expressions and their solution	Lotto	CM	page 74
	Magic Square	C2	page 82
	Pyramid Construction	C2	page 87
	Hidden Exercises	GI	page 95
	Mathematical Memory	PG	page 114
Integral numbers, operations with integral numbers	Lotto	CM	page 74
	Magic Square	C2	page 82
	Pyramid Construction	C2	page 87
	Hidden Exercises	GI	page 95
	Mathematical Memory	PG	page 114

THEMATIC AREA	NAME AND TYPE OF DIDACTIC GAME: CM = cooperative game for multiple teams C2 = cooperative game for teams of 2 GI = games for individuals PG = pair games		
Divisibility of positive integers	Magic Square	C2	page 82
	The Way Home	GI	page 90
	Bard	PG	page 99
	Dim	PG	page 102
	Number of Divisors	PG	page 121
Decimal numbers, operations with decimal numbers	Lotto	CM	page 74
	Magic Square	C2	page 82
	Pyramid Construction	C2	page 87
	Hidden Exercises	GI	page 95
	Domino	PG	page 104
	Mathematical Memory	PG	page 114
Functions	Lotto	CM	page 74
	Treasure Seekers	PG	page 111
Combinatorics	Logic	PG	page 113
Powers and Roots	Hidden Exercises	GI	page 95
	Enades	PG	page 106
	Powers	PG	page 117

THEMATIC AREA	NAME AND TYPE OF DIDACTIC GAME: CM = cooperative game for multiple teams C2 = cooperative game for teams of 2 GI = games for individuals PG = pair games		
Volume and area of geometric shapes	Builders	CM	page 63
	Cube with Letters	C2	page 80
Area of geometric shapes	Builders	CM	page 63
	Lotto	CM	page 74
Perimeter of geometric shapes	Builders	CM	page 63
Percentage	Lotto	CM	page 74
Conversions of units	Domino	PG	page 104
Rational numbers, operations with rational numbers	Lotto	CM	page 74
	Magic Square	C2	page 82
	Pyramid Construction	C2	page 87
	Hidden Exercises	GI	page 95
	Domino	PG	page 104
	Mathematical Memory	PG	page 114

THEMATIC AREA	NAME AND TYPE OF DIDACTIC GAME: CM = cooperative game for multiple teams C2 = cooperative game for teams of 2 GI = games for individuals PG = pair games		
Solution of linear equations and their systems	Lotto	CM	page 74
	Equations	PG	page 123
Parallelogram, trapezoid	Jigsaw	C2	page 85
Spatial imagination development	Cube with Letters	C2	page 80
	3D Noughts and Crosses	PG	page 98
Central and axial symmetry	Symmetric Pictures	GI	page 93
	Snake	PG	page 108
	Letter 'L' Travelling	PG	page 119
Triangle	Jigsaw	C2	page 85
Angle and its dimensions	Jigsaw	C2	page 85

CONCLUSION

The only way to learn
mathematics is to 'do' mathematics.
P. R. Halmos, (1982)

In its 6 chapters, the presented book has given an overview of didactic games and their place in mathematics teaching both from the theoretical and practical point of view.

We believe that our publication has provided sufficient information about didactic games in teaching math. We also dare to hope that it has aroused readers' interest in this issue, as well as inspired them to try playing the mentioned didactic games or try teaching with them in practice. If we have succeeded in this, our book has fulfilled its goal.

If you have any questions regarding didactic games and their use in mathematics teaching, please contact us at peter.vankus@gmail.com. We will also be glad if you share with us all your experience with the practical use of didactic games in mathematics education.

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Didactic Games in Mathematics

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